Analysis and Interpretation of Astronomical Spectra

Theoretical Background and Practical Applications for Amateur Astronomers

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1 Introduction

Technological advances like CCD cameras, but also affordable spectrographs on the market cause a significant upturn of spectroscopy within the community of amateur astronomers. Further freeware programs and detailed instructions are available to enable the processing, calibrating and normalising of the spectra. Several publications explain the function and even the self-construction of spectrographs and further many papers can be found on specific monitoring projects. The numerous possibilities however for analysis and interpretation of the spectral profiles, still suffer from a considerable deficit of suitable literature.

This publication presents as a supplement practical applications and the appropriate astrophysical backgrounds. Further the Spectroscopic Atlas for Amateur Astronomers [33] is available, which covers all relevant spectral classes by commenting most of the lines, visible in medium resolved spectral profiles. It is primarily intended to be used as a tool for the line identification. Each spectral class, relevant for amateurs, is presented with their main characteristics and typical features.

Further, a detailed tutorial [30] (German only) on the processing of the spectra with the Vspec and IRIS software is downloadable. Since all documents should remain independent, some text and graphics are included redundantly.

Spectroscopy is the real key to astrophysics. Without them, our current picture of the universe would be unthinkable. The photons, which have been several million years "on the road" to our CCD cameras, provide an amazing wealth of information about the origin object. This may be fascinating, even without the ambition to strive for academic laurels. Further there is no need for a degree in physics with, specialisation in mathematics, for a rewarding deal with this matter. Required is some basic knowledge in physics, the ability to calculate simple formulas with given numbers on a technical calculator and finally a healthy dose of enthusiasm.

Also the necessary chemical knowledge remains very limited. In the hot stellar atmospheres and excited nebulae the individual elements can hardly undergo any chemical compounds. Only in the outermost layers of relatively "cool" stars, some very simple molecules can survive. More complex chemical compounds are found only in really cold dust clouds of the interstellar space and in planetary atmospheres – a typical domain of radio astronomy. Moreover in stellar astronomy, all elements, except hydrogen and helium, are simplistically called as "metals".

The share of hydrogen and helium of the visible matter in the cosmos is still about 99%. The most "metals", have been formed long time after the Big Bang within the first generation of massive stars, which distributed it in to the surrounding space by Supernova explosions at the end of their live.

Much more complex, however, is the quantum-mechanically induced behavior of the excited atoms in stellar atmospheres. These effects are directly responsible for the formation and shape of the spectral lines. Anyway for the practical work of the "average amateur" some basic knowledge is sufficient.
2 Photons – Messengers from the Universe

2.1 Photons transport Information and Energy

Photons are generated in stars, carrying its valuable information over immense periods of time and unimaginable distances, and finally end in the pixel field of our CCD cameras. By their “destruction” they deposit the valuable information, contributing electrons to the selective saturation of individual pixels – in fact trivial, but somehow still fascinating. By switching a spectrograph between the telescope and camera the photons will provide a wealth of information which surpasses by far the simple photographic image of the object. It is therefore worthwhile to make some considerations about this absolutely most important link in the chain of transmission.

It was on the threshold of the 20th Century, when it caused tremendous "headaches" to the entire community of former top physicists. This intellectual "show of strength" finally culminated in the development of quantum mechanics. The list of participants reads substantially like the Who's Who of physics at the beginning of the 20th century: Werner Heisenberg, Albert Einstein, Erwin Schrödinger, Max Born, Wolfgang Pauli, Niels Bohr, just to name a few. Quantum mechanics became, besides the theory of relativity, the second revolutionary theory of the 20th Century. For the rough understanding about the formation of the photons and finally of the spectra, the necessary knowledge is reduced to some key points of this theory.

2.2 The Duality of Waves and Particles

Electromagnetic radiation has both wave and particle nature. This principle applies to the entire spectrum. Starting with the long radio waves, it remains valid on the domains of infrared radiation, visible light, up to the extremely short-wave ultraviolet, X-rays and gamma rays.


For our present technical applications, both properties are indispensable. For the entire telecommunications, radio, TV, mobile telephony, as well as the radar and the microwave grill it's the wave character. The CCD photography, light meter of cameras, gas discharge lamps (eg energy saving light bulbs and street lighting), and last but not least, the spectroscopy would not work without the particle nature.

2.3 The Quantisation of the Electromagnetic Radiation

It was one of the pioneering discoveries of quantum mechanics that electromagnetic radiation is not emitted continuously but rather quantised (or quasi "clocked"). Simplified explained a minimum "dose" of electromagnetic radiation is generated, called "photon", which belongs to the Bosons within the "zoo" of elementary particles.
2.4 Properties of the Photons

- Without external influence photons have an infinitely long life.
- Their production and “destruction” takes place in a variety of physical processes. Relevant for the spectroscopy are electron transitions between different atomic orbital (details see later).
- A photon always moves with light speed. According to the Special Relativity Theory (SRT) it can therefore possess no rest mass.
- Each photon has a specific frequency (or wavelength), which determines its energy – the higher the frequency, the higher the energy of the photon (details see sect. 10).
3 The Continuum

3.1 Black Body Radiation and the Course of the Continuum Level

The red curve, hereafter referred to as continuum level $I_c$, corresponds to the course of the radiation intensity, plotted over the wavelength (increasing from left to right). It is cleaned and smoothed by any existing absorption or emission lines (blue curve). The entire area between the horizontal wavelength axis and the continuum level is called continuum [5].

Most important physical basis for the origin and course of the continuum is the so-called black body radiation. The blackbody is a theoretical working model which in that perfection doesn’t exist in nature.

For most amateurs it is sufficient to know, that:

- The blackbody is an ideal absorber which absorbs broadband electromagnetic radiation, regardless of the wavelength, completely and uniformly.
- The ideal black body represents a thermal radiation source, which emits a broad-band electromagnetic radiation, according to the Planck's radiation law, with an exclusively temperature-dependent intensity profile.
- Stars in most cases may simplified be considered as black-body radiators.

3.2 Wien's Displacement Law

This theory has practical relevance for us because the intensity profile of the spectrum provides information about the temperature of the radiator! The radiation distribution of different stars shows bell-shaped curves, whose peak intensity shifts to shorter wavelength, respectively higher frequency with increasing temperature (Planck's radiation law).

Quelle: www.astro.umass.edu

Wilhelm Wien
With Wien’s displacement law (German physicist Wilhelm Wien 1864-1928) and the given wavelength $\lambda$ [Å] of the maximum radiation intensity, it is theoretically possible to calculate the star temperature $T$ [K].

[Å]: Angström, $1$ Å $= 10^{-10}$ m  
[K]: Kelvin $K = °Celsius + 273°$

$$\lambda[Å] = \frac{28'978'200}{T} \quad \{1\} \quad T[K] = \frac{28'978'200}{\lambda} \quad \{2\}$$

Examples:  
Alnitak  $T = \text{ca. 25'000 K}$  $\lambda_{\text{Tmax}} = 1'160$ Å (Ultraviolet)  
Sun  $T = \text{ca. 5'500 K}$  $\lambda_{\text{Tmax}} = 5'269$ Å (Green)  
Betelgeuse  $T = \text{ca. 3'450 K}$  $\lambda_{\text{Tmax}} = 8'400$ Å (Infrared)

### 3.3 The Pseudo Continuum

By all stellar spectra, the course of the unprocessed continuum differs strongly from the theoretical shape of reference curves, regardless if recorded with professional or amateur equipments. The reasons are primarily interstellar, atmospheric and instrument-specific effects (telescope, spectrograph, camera), which distort the original intensity course of the spectral profile to a so called pseudo continuum. Therefore, the Wien’s displacement law, on the basis of the maximum profile intensity, can be observed only qualitatively. The following chart shows the superimposed spectral profiles (pseudo continua) of all bright Orion stars, obtained with a simple transmission grating (200L/mm), a Canon compact camera (Powershot S 60) and processed with the Vspec software. Denoted are here the spectral classes, as well as some identified absorption lines.

Here it is obvious, that the profile shapes and their maximum intensities of the late O- and early B-classes (sect. 13) are nearly identical. As expected, this intensity is by Rigel, a slightly lesser hot, late-B giant (green profile), and in stark extent by the cool M-giant Betelgeuse (orange profile), shifted to the right towards larger wavelengths. Theoretically, the maximum intensities of the O and B stars would be far left, outside of the diagram in the UV range, for the cold Betelgeuse far right and also outside the diagram in the IR range. Main causes for this error are the spectral selectivity of the CCD chip and the IR filter in the compact camera, pretending that all the peaks would be within the diagram. Here is also clearly visible, that the absorption lines (sect. 5.2) are quasi “imprinted” on the continuum profile, similar to the modulation on a carrier frequency.
These lines carry the information about the object, the course of the continuum reveals only the temperature of the radiator. The profile of Betelgeuse shows impressively, that the spectra of cool stars are dominated by broad molecular titanium oxide (TiO) bands (sect. 5.4). The example also shows the dramatic influence of the spectral characteristics of the camera. In the blue wavelength range, the sensitivity of most cameras drops quickly. Astronomical cameras usually have easy removable/upgradable IR filters, without spectra can be recorded well in to the IR range.
4 Spectroscopic Wavelength Domains

4.1 The Usable Spectral Range for Amateurs

The professional astronomers study the objects nowadays in nearly the entire electromagnetic spectrum – including also radio astronomy. Also space telescopes are used, which are increasingly optimised for the infrared region in order to record the extremely red-shifted spectra of objects from the early days of the universe (sect. 15.5–15.8). For the ground-based amateur, equipped with standard telescopes and spectrographs only a modest fraction of this domain is available. The usable range for us is, in addition to the specific design features of the spectrograph, limited mainly by the spectral characteristics of the camera including any filters. The Meade DSI III e.g. achieves with the DADOS spectrograph useful results in the range of approximately 3800 – 8000 Å, i.e. throughout the visible domain and the near infrared part of the spectrum. Here also the best known and best documented lines are located, such as the hydrogen lines of H-Balmer series and the Fraunhofer lines (see later).

4.2 The Selection of the Spectral Range

For high-resolution spectra, the choice of the range is normally determined by a specific monitoring project or the interest in particular lines. Perhaps also the calibration lamp emission lines have to be considered in the planning of the recorded section.

For low-resolution, broadband spectra mostly the range of the H-Balmer series is preferred (sect. 9). Hot O- and B- stars can be taken rather in the short-wave part, because their maximum radiation lies in the UV range. It usually makes little sense to record the area on the red side of H$\alpha$, except the emission lines of P Cygni and Be stars. Between approximately 6,200 – 7,700Å (see picture below), it literally swarms of atmospheric related (telluric) H$_2$O and O$_2$ absorption bands.

Apart from their undeniable aesthetics they are interesting only for atmospheric physicist. For astronomers, they are usually only a hindrance, unless the fine water vapour lines are used to calibrate the spectra! There are features in the Vspec and similar software, to attenuate such terrestrial absorption bands.

By the late spectral types of K, and the entire M-Class (sect. 13.5), however, it makes sense to record this range, since the radiation intensity of these stars is very strong in the IR range and shows here particularly interesting molecular absorption bands. Also, the reflection spectra (sect. 5.7) of the large gas planets show mainly here the impressive gaps in the continuum. Useful guidance for setting the wavelength range of the spectrograph is the micrometer scale, the calibration lamp spectrum or the daylight (solar) spectrum, respectively. At night the reflected solar spectrum is available from the moon and the planets. A good marker on the blue side is the impressive double line of the Fraunhofer H- and K Absorption (sect. 13.2.).
4.3 Terminology of the Spectroscopic Wavelength Domains

Terminology for wavelength domains is used inconsistently in astrophysics [4] and depends on the context. Furthermore many fields of astronomy, various satellite projects etc. often use different definitions.

Here follows a summary according to [4] and Wikipedia (Infrared Astronomy). Given are either the center wavelength \( \lambda \) of the corresponding photometric band filters, or their approximate passband.

**Optical range UBVRI**  \( \lambda \lambda 3,300 - 10,000 \) (Johnson/Bessel/Cousins)

<table>
<thead>
<tr>
<th>Center wavelength ( \lambda ) [( \mu m )]</th>
<th>Astrophysical wavelength domain</th>
<th>Required instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35 3,500</td>
<td>U – Band (UV)</td>
<td>Most optical telescopes</td>
</tr>
<tr>
<td>0.44 4,400</td>
<td>B – Band (blue)</td>
<td></td>
</tr>
<tr>
<td>0.55 5,500</td>
<td>V – Band (green)</td>
<td></td>
</tr>
<tr>
<td>0.65 6,500</td>
<td>R – Band (red)</td>
<td></td>
</tr>
<tr>
<td>0.80 8,000</td>
<td>I – Band (infrared)</td>
<td></td>
</tr>
</tbody>
</table>

Further in use is also the \( Z \)–Band, some \( \lambda \lambda 8,000 – 9,000 \) and the \( Y \)–Band, some \( \lambda \lambda 9,500 – 11,000 \) (ASAHI Filters).

**Infrared range** according to Wikipedia (Infrared Astronomy)

<table>
<thead>
<tr>
<th>Center wavelength ( \lambda ) [( \mu m )]</th>
<th>Astrophysical wavelength domain</th>
<th>Required instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25 10,250</td>
<td>J – Band</td>
<td>Most optical- and dedicated infrared telescopes</td>
</tr>
<tr>
<td>1.65 16,500</td>
<td>H – Band</td>
<td></td>
</tr>
<tr>
<td>2.20 22,000</td>
<td>K – Band</td>
<td></td>
</tr>
<tr>
<td>3.45 34,500</td>
<td>L – Band</td>
<td>Some optical- and dedicated infrared telescopes</td>
</tr>
<tr>
<td>4.7 47,000</td>
<td>M – Band</td>
<td></td>
</tr>
<tr>
<td>10 100,000</td>
<td>N – Band</td>
<td></td>
</tr>
<tr>
<td>20 200,000</td>
<td>Q – Band</td>
<td></td>
</tr>
<tr>
<td>200 2,000,000</td>
<td>Submilimeter</td>
<td>Submilimeter telescopes</td>
</tr>
</tbody>
</table>

For ground based telescopes mostly the following terminology is in use [Å]:

- Far Ultraviolet (FUV): \( \lambda <3000 \)
- Near Ultra Violet (NUV): \( \lambda 3000 – 3900 \)
- Optical (VIS): \( \lambda 3900 – 7000 \)
- Near Infrared (NIR): \( \lambda 6563 (H\alpha) – 10,000 \)
- Infrared or Mid-Infrared: \( \lambda 10,000 – 40,000 \) (J, H, K, L – Band 1 – 4 \( \mu m \))
- Thermal Infrared: \( \lambda 40,000 – 200,000 \) (M, N, Q – Band 4 - 20\( \mu m \))
- Submilimeter: \( \lambda >200,000 \) (200 \( \mu m \))
5 Typology of the spectra

5.1 Continuous Spectrum

Incandescent solid or liquid light sources emit, similar to a black body radiator, a continuous spectrum, e.g., bulbs. The maximum intensity and the course of the continuum obey the Plank's radiation law.

5.2 Absorption Spectra

An absorption spectrum is produced when radiated broadband light has to pass a low pressure gas layer on its way to the observer. Astronomically, the radiation source is in the majority of cases a star and the gas layer to be traversed is its own atmosphere. Depending on the chemical composition of the gas, it will absorb photons of specific wavelengths by exciting the atoms, i.e., single electrons are momentarily lifted to a higher level. The absorbed photons are ultimately lacking at these wavelengths, leaving characteristic dark gaps in the spectrum, the so-called absorption lines. This process is described in more detail in sect. 9.1. The example shows absorption lines in the green region of the solar spectrum (DADOS 900L/mm).

5.3 Emission Spectra

An emission spectrum is generated when the atoms of a thin gas are heated or excited so that photons with certain discrete wavelengths are emitted, e.g., neon glow lamps, energy saving lamps, sodium vapor lamps of the street lighting, etc. Depending on the chemical composition of the gas, the electrons are first raised to a higher level by thermal excitation or photons of exactly matching wavelengths – or even completely released, where the atom becomes ionised. The emission takes place after the recombination or when the excited electron falls back from higher to lower levels, while a photon of specific wavelength is emitted (sect. 9.1). Astronomically, this type of spectral line comes mostly from ionised nebulae (sect. 21) in the vicinity of very hot stars, planetary nebulae, or extremely hot stars, pushing off their gaseous envelops (e.g., P Cygni). The following figure shows an emission spectrum (Hα, Hβ, [O III]) of the Orion Nebula M42, which is ionised by its central, very hot Trapezium stars θ1 Orionis. The dark absorption lines stem from θ1 Orionis or O2, respectively H2O molecules in the earth’s atmosphere. (DADOS Spectrograph, 200L/mm).
5.4 Band Spectra

Band spectra are caused by highly complex rotational and vibrational processes by heated molecules. Such are generated in the relatively cool atmospheres of red giants. But also several of the prominent Fraunhofer lines in the solar spectrum (sect. 13.2) are caused by molecular absorption, such as the striking A-Band due to O$_2$ in the earth’s atmosphere.

The following spectrum originates from Betelgeuse (DADOS 200L/mm). It shows only a few discrete lines. The majority is dominated by absorption bands, which are here mainly caused by titanium oxide (TiO) and to a lesser extent by magnesium hydride (MgH).

In this case, these asymmetric structures reach the greatest intensity on the left, short-wave band end (called bandhead) and then slowly weakening to the right. The wavelength of absorption bands always refers to the point of greatest intensity ("most distinct edge").

The following diagram shows CO absorption bands (carbon monoxide) in the infrared spectrum of 51 Ophiuchi (1micron or $\mu$m $=10'000\text{Å}$). Generally by some carbon molecules (eg CO, C$_2$) the intensity gradient of the absorption bands runs in the opposite direction as by Titan Oxide. Already in the middle of the 19th Century this effect has been recognised by Father Secchi (Sect. 13.3). For such spectra, he introduced the “Spectral type IV”.

(source: http://www.aanda.org/ Evidence for a hot dust-free inner disk around 51 Oph.)
5.5 Mixed Emission- and Absorption Spectra

There are many cases where absorption and emission lines appear together in the same spectrum. The best known example is P Cygni, a textbook object for amateurs. To this unstable and variable supergiant of the spectral type B2 Ia numerous publications exist. In the 17th Century, it appeared for 6 years as a star of the third magnitude, and then "disappeared" again. In the 18th Century it gained again luminosity until it reached its current, slightly variable value of approximately +4.7" to +4.9". The distance of P Cygni is estimated to ca. 5000 – 7000 ly (Karkoschka 5000 ly).

The picture below shows the expanding shell, taken with the Hubble Space Telescope (HST). The star in the center is fully covered. The diagram right shows the typical formation of the so-called P Cygni profiles, which are shown here in the violet region of the spectrum (DADOS 900L/mm).

In the area of the blue arrow a small section of the shell, consisting of thin gas, is moving exactly toward Earth and generating blue-shifted absorption lines (Doppler Effect). The red arrows symbolise the light, emitted by sections of the shell, expanding sideward, producing emission lines. In the combination results a broad emission line and a generally less intense blue-shifted absorption line. P Cygni profiles are present in almost all spectral types and are a reliable sign of a massive radial motion of matter ejected from the star.

Based on the wavelength difference between the absorption and emission part of the line, the expansion velocity of the envelope can be estimated using the Doppler formula (sect. 15). This object is further described in sect. 17, where also the estimation of the expansion velocity is demonstrated.

5.6 Composite Spectra

Superimposed spectra of several light sources are also called composite spectra. The English term was coined in 1891 by Pickering for composite spectra in binary systems. Today it is often used also for integrated spectra of galaxies and quasars, which consist up to several hundred billion superposed individual spectra.
5.7 Reflection Spectra

The objects of our solar system are not self-luminous, but only visible thanks to reflected sunlight. Therefore, these spectra always contain the absorption lines of the solar spectrum. The continuum course is however coined, because certain molecules in the atmospheres of the large gas planets, e.g., CH₄ (methane), absorb and/or reflect the light differently strong at specific wavelengths.

The following chart shows the reflection spectrum of Jupiter (red), recorded with the DA-DOS spectrograph and the 200L/mm grating. Superimposed (green) is previously in the dawn light captured daylight (solar) spectrum. Both profiles have been normalised on the same continuum section [30]. In this wavelength range, the most striking intensity differences are between 6100 and 7400 Å.
6 Form and Intensity of the Spectral Lines

6.1 The Form of the Spectral Line

The chart on the right shows several absorption lines with the same wavelength, showing an ideal Gaussian-like intensity distribution but different width and intensity. According to their degree of saturation, they penetrate differently deep into the continuum, maximally down to the wavelength axis. The red profiles are both unsaturated. The green one, which just touches the deepest point on the wavelength axis, is saturated and the blue one even oversaturated [5].

The lower part of the profile is called "Core", which passes in the upper part over the "Wings" in to the continuum level. The short-wavelength wing is called "Blue Wing", the long-wave- "Red Wing" [5]. In contrast to the presented absorption lines, emission line profiles rise always upwards from the continuum level.

6.2 The Information Content of the Line Shape

There exists hardly a stellar spectral line, which shows this ideal shape. But in the deviation from this form a wealth of information about the object is hidden. Here are some examples of physical processes which have a characteristic influence on the profile shape and become therefore measurable:

- The rotational speed of a star flattens and broadens the line (rotational broadening), see sect. 16.
- Temperature and density of the stellar atmosphere broaden the line (temperature / pressure/collision broadening) see sect. 13.9.
- Macro Turbulences in the Stellar Atmosphere broaden the line, see sect. 16.6
- Instrumental responses broaden the line (instrumental broadening)
- In strong magnetic fields (eg sunspots) a splitting and shifting of the spectral line occurs due to the so-called Zeeman Effect.
- Electric fields produce a similar phenomenon, the so-called Stark Effect.

6.3 Blends

Stellar spectral lines are usually more or less strongly deformed by closely neighbouring lines - causing this way so-called "blends".

6.4 The Fully Saturated Absorption Line in the Spectral Diagram

The spectral profile is generated based on to the course of the gray values within the spectral stripe of the processed CCD image. The possible range from black to white, covered by Vspect, comprises 256 gray levels [411]. With the moderately high resolution of the DADOS spectrograph in the astronomical field no fully saturated absorption lines can be observed. This provides not even the impressive Fraunhofer A-line (sect. 13.2). For demonstration
purposes, I have therefore generated a Vspec intensity profile (blue) from black to white, based on an 11-step gray-scale chart. The Profile section in the black area is here, as expected, saturated to 100% and runs therefore on the lowest level, i.e., congruent with the wavelength axis. The saturation of the remaining gray values decreases staircase-like upward, until it finally becomes white on the continuum level. If an underexposed spectral stripe was prepared in advance with IRIS [410] [30], the gray scale is stretched, so that the highest point on the chart becomes white. Thus, a maximum contrast is achieved.

6.5 The Fully Saturated Emission Line in the Spectral Diagram

No tricks are required for the presentation of an *oversaturated* emission line. This just needs to overexpose the calibration lamp spectrum. Such oversaturated Neon lines appear flattened on the top. Such an unsuccessful Neon spectrum must never be used for calibration purposes!
The measurement of the spectral lines

The measurements, described in the following, are mostly supported by the spectroscopic software (e.g., Vspec) and usually made by marking the lines with the cursor [411], [30].

7.1 The Peak Intensity \( P \)

The peak intensity offers the easiest way to measure the intensity of a spectral line. The line depth, directly measured in a non-normalized profile, is not really a true measure of the intensity. Only considered in relation to the local continuum level \( I_c \), the peak intensities get comparable to each other. This applies to both, absorption- and emission lines.

\[ P = \frac{I}{I_c} \quad \{14\} \]

7.2 FWHM Full Width at Half Maximum Height

The FWHM value is the line width in [Å] at half height of the maximum intensity. It can be correctly measured even in non-normalized spectral profiles. The width of a spectral line is inter alia depending on temperature, pressure, density, and turbulence effects in stellar atmospheres. It allows therefore important conclusions and is often used as a variable in equations, e.g., to determine the rotational velocity of stars (sect. 16.6).

This line width is specified in most cases as wavelength-difference \( \Delta \lambda \). For the measurement of rotational and expansion velocities \( FWHM \) is also expressed as a velocity value according to the Doppler principle. For this purpose \( FWHM \) [Å] is converted with the Doppler formula \( \{15\} \)

\[ v = FWHM \cdot c/\lambda \text{ to a speed value [km/s] (sect. 15).} \]

The FWHM value, obtained from the spectrum [30] has now to be corrected from the instrumental broadening.

\[ FWHM_{korr} = \sqrt{FWHM_{measured}^2 - FWHM_{instrument}^2} \quad \{3\} \]

\( FWHM_{instrument} \) corresponds to the theoretical maximum resolution \( \Delta \lambda_S \) [Å] of the spectrograph, i.e., the smallest dimension of a line detail, which can be resolved.

The resolution is limited on one side by the optical design of the spectrograph (dispersion of the grating, collimator optics, slit width, etc.). It can mostly be found in the manual of the spectrograph as so-called \( R \)-Value \( R = \lambda/\Delta \lambda_S \) which is valid for a defined wavelength range (\( \lambda \) = considered wavelength).

\[ R = \frac{\lambda}{\Delta \lambda_S} \quad \{4\} \quad FWHM_{instrument} = \frac{\lambda}{R} \quad \{4a\} \]

It is generally suggested (e.g., [123]) to determine this value at the emissions of a calibration lamp spectrum by several averaged measurements of the FWHM. The resolution can also be limited by the pixel grid of the connected camera [Å/pixel], if this value is greater than \( \Delta \lambda_S \) of the spectrograph. For a wavelength-calibrated profile, this value is shown in the head panel of the Vspec screen. For color cameras an additional loss of resolution is to take into account, compared to a black and white chip with the same pixel grid.
7.3 **EW, Equivalent Width**

The *Equivalent Width* is a combined measure for the *width and intensity* of a spectral line. The profile area between the continuum level $I_c$ and the profile of the spectral line has the same size as the rectangular area with the fully saturated depth (here $I_c = 1$) and the equivalent width $EW$ [Å].

$$EW = \frac{\text{Profile area}}{I_c} \quad \{5\}$$

The $EW$-value must therefore be measured in a spectrum, normalised to $I_c = 1$ ([30], sect. 10). This is the mathematically correct expression for $EW$:

$$EW = \int_{\lambda_1}^{\lambda_2} \frac{I_c - I_\lambda}{I_c} d\lambda \quad \{5a\}$$

In simple terms the red area above the spectral curve is calculated by summing up an infinite number of vertical, infinitely narrow rectangular strips with the width $d\lambda$ and the variable heights $I_c - I_\lambda$, within the entire range from $\lambda_1$ to $\lambda_2$. To get finally the equivalent width $EW$, these values are still to divide by the entire *height* $I_c$ of the continuum-, resp. of the saturated square. The integral sign $\int$ is derived from the letter S, and stands here for "sum". $I_c$ is the continuum intensity, $I_\lambda$ the variable intensity of the spectral line depending on (or a function of) the wavelength $\lambda$.

$EW$ values of absorption lines are by definition *always positive* (+), those of emission lines *negative* (−).

Since the $EW$ value is always measured at a continuum level, normalized to $I_c = 1$, it is *neither* influenced by the *course* of the continuum, *nor* by the absolute radiation flux. Should $EW$ be measured in a *non rectified* profile, the continuum must be normalised *immediately at the base* of the spectral line to $I_c = 1$!

In scientific publications $EW$ is also designated with the capital $W$. $W_\alpha$ eg designates the equivalent width of the H$\alpha$ Line. Even more confusing, − in some publications I have also found the FWHM value expressed in $W$. The conclusion: One must always simply check which variable is really meant.

7.4 **FWZI** Full Width at Zero Intensity

Rather rarely the FWZI value of a spectral line is needed. The *Full With at Zero Intensity* corresponds to the integration range according to formula and chart {5a}:

$$FWZI = \lambda_2 - \lambda_1 \quad \{5b\}$$

7.5 **Normalised Equivalent Width $W_\lambda$**

Rather rarely the normalised $EW$ value $W_\lambda$ is used [128]:

$$W_\lambda = \frac{EW}{\lambda} \quad \{6\}$$

This allows the comparison of EW-values of different lines at *different wavelengths* $\lambda$, taking into consideration the linearly increasing photon energy towards decreasing wavelength $\lambda$ [9]. Anyway, for most empirical formulas and procedures this is not applied.
7.6 Influence of the Spectrograph Resolution on the FWHM and EW Values

The above outlined theories about \textit{FWHM}- and \textit{EW} must realistically be relativated. This need is dramatically illustrated by the following spectral profiles of the Sun, taken with different highly resolving spectrographs (M. Huwiler/R. Walker). The R-values are here within a range of approximately \(800 – 80,000\).

**Sun Spectrum \(\lambda 5256 – 5287 \text{ Å}\)**
Comparison Prototype Echelle- with Cerny Turner Spectrograph

**Sun Spectrum \(\lambda 5160 – 5270 \text{ Å}\)**
Comparison Prototype Echelle- with DADOS Spectrograph 900- and 200 L mm\(^{-1}\)

Magnesium Triplet: \(\lambda 5167, 5173, 5183 \text{ Å}\)
The comparison of these graphs shows the following:

• If the resolution (R) is increased it becomes clearly evident that in stellar spectra practically no "pure" lines exist. Apparent single lines almost turn out as a "blend" of several sub lines, if considered at higher resolutions.

• Striking is also the so-called "Instrumental Broadening". Even relatively well-insulated seeming lines broaden dramatically with decreasing resolution (R), due to the instrumental influences. This affects both, the measured FWHM or the half-width of a line, as well as the EW value, which refers to the area of the profile.

• As a result of the "Blend problem" FWHM and EW values, obtained with significantly different resolutions (R value), are not directly comparable.

Conclusion:

• FWHM-/EW values are only seriously comparable if they have been obtained from profiles with similar resolution. This necessarily requires a declaration of the resolution (R-value).

• This issue concerns very intense and relatively isolated lines, to a somewhat lesser extent (eg Ha, limited to the earlier spectral classes). Anyway, even here the FWHM values require necessarily the reduction of the "Instrumental Broadening" with the formulas \[3\] \[4\]. Measurements at Gaussian fitted, intense spectral lines of the H-Balmer series, taken with Vspect at resolutions of \(R \approx 800\) and \(R \approx 4000\), have shown in some cases significantly different EW values.

• For the EW values the section of the continuum must be specified, on which the profile is normalised. This is unavoidable at least for the late spectral classes K and M, which exhibit a saw tooth-shaped profile due to molecular absorption of titanium oxide, and therefore a very diffuse continuum.

• EW and FWHM-values must always be measured on a Gaussian fit, since the profile shape is mostly distorted by blends. The same applies in most cases also for the wavelength, ie the barycenter of the profile area. At extremely deformed lines, the measurement of \(\lambda\) at the maximum peak intensity may also make sense, depending on the specific application.

• For monitoring projects in the amateur field, it is important that all participants work with similarly high resolutions and the recording and processing of the spectra is clearly standardised.
8 Calibration and Normalisation of Spectra

There are several ways to calibrate and normalise spectra. The practical execution is demonstrated in [411], [30].

8.1 The Calibration of the Wavelength

Because spectra are plotted as course of the radiation intensity over the wavelength, two dimensions can be calibrated. For most applications, only the calibration of the wavelength is required. This can be done relatively easy with lines of known wavelength or spectral calibration lamps [30].

8.2 The Pseudo Continuum

In most cases the course of the pseudo continuum is anyway useless, because the original radiation characteristic of the star is massively distorted by interstellar, atmospheric and instrumental influences. Depending on the wavelength it just shows the amount of electrons, which has been read out, amplified by the camera electronics and finally averaged by the spectral processing software over a defined height of the vertical pixel rows. Thus it reflects roughly proportional also the recorded photon flux. As intensity unit for raw profiles therefore often ADU (Analog Digital Units) is used.

8.3 The Removal of the Pseudo Continuum

The easiest and in most cases adequate solution is to remove or "rectify" [4] the pseudo continuum by dividing it by its own course. After this division, the profile runs horizontally and the intensity of spectral lines is this way automatically adjusted in proportion to their original $I_c$ value.

This rectified profile enables now on a glance a rough relative comparison between the intensities of individual spectral lines. This is impressively demonstrated in the solar spectrum by the two Fraunhofer H- and K lines of ionised calcium (Ca II). The blue profile of the pseudo continuum shows these lines only stunted at the short wavelength end of the spectrum (blue arrow). After the removal of the continuum (red profile), the H- and K lines appear now obviously as the strongest absorptions, which the sun generates itself (red arrow).

Important: The Division of the pseudo continuum by any correction curves changes exclusively the course of the continuum. Neither the Peak Intensity $P = I/I_c$ nor the Equivalent width $EW$, are therefore affected by the specific course of the continuum or by the absolute intensity of the continuum radiation at this wavelength!

The $P$ and $EW$ values of emission lines may be distorted by the "interstellar reddening" (frequency dependent). This error can’t be eliminated simply by dividing by correction curves, but only with relatively complex corrective algorithms. This process is presented in sect. 20.
8.4 The Relative Radiometric Correction of the Profile

This method fits the pseudo continuum to the synthetically generated, theoretical course of the radiation intensity of a standard star. Similar to sect. 8.3 this process automatically adjusts also the intensity of the spectral lines proportional to the intensity of the original $I_c$ values and thus neither the $I/I_c$ ratios, nor the equivalent widths $EW$ of the individual spectral lines are modified.

This way we obtain a spectral profile, whose continuum course is closest possible to reality and can be found in this form in professional spectral databases. Except for educational purposes this is of little practical value for amateur applications, and shows similar disadvantages for the line intensity comparison like the pseudo continuum. Further this relative correction doesn’t yet scale the intensity axis in physical units! In the following chart the blue profile is the pseudo continuum of Sirius. The red profile is the reference curve of a synthetic standard star from the same spectral class (Spectral library Vspec). It is smoothed and cleaned from the spectral lines (Vspec).

The division of the blue pseudo continuum by the red reference continuum results in the green curve called "instrumental response". Finally the division of the blue pseudo continuum by the green instrumental response profile results in the radiometrically corrected profile (black). It shows now the same continuum shape like the red smoothed reference profile.

For correction curves, obtained this way, the familiar term "instrumental response" is misleading. This curve doesn’t just reflect the erroneous recording characteristics of the spectrograph, camera and telescope, but also the wavelength dependent filtering effect of the Interstellar Matter and of the Earth's atmosphere. Strongly different spectral classes with
distinctly different temperatures must therefore inevitably generate strongly different correction curves. Therefore the often postulated “all purpose use” of just one single correction curve causes strong errors, if used for other spectral classes. The diagram shows for three different spectral classes three totally different response curves (Vspec).

The already previously presented Sirius spectrum here finally demonstrates that the absorption lines in the raw spectrum (blue) exhibit the same relative depth \( I/I_c \) into the continuum as their counterparts in the radiometrically corrected profile (black). Anyway the absolute sizes of the individual lines \( I \) are very different. This manifests itself most strikingly in the range of the short-wave H-Balmer lines.

The equivalent widths \( EW \) also remain unchanged, since these values are measured anyway at a continuum level, normalised to \( I_c = 1 \) and thus the integrated profile areas remain the same (according to formula (5a)).

### 8.5 The Intensity Comparison between Different Spectral Lines

Thus, the intensities of different lines may be compared with both, the peak intensities \( P \) as well as the equivalent widths \( EW \). In practice the equivalent widths are preferred because they provide more accurate results.

\[
\frac{P_1}{P_2} = \frac{EW_1}{EW_2}
\]

For special cases, see also sect. 7.4, the normalised \( EW \) value \( W_2 \).
8.6 The Absolute Flux Calibration
As a final step of the radiometric profile correction, the intensity axis can be absolutely flux-calibrated in physical units, for spectra in the optical range usually in [erg s\(^{-1}\) cm\(^{-2}\) Å\(^{-1}\)]. This intensity calibration of a raw profile is demanding and very time consuming. It is performed with an absolute flux calibrated profile of a standard star of similar spectral class and recorded with the same elevation angle as the analysed star. For this correcting procedure even the exposure time of the recorded spectra is needed. In the professional sector also the atmospheric extinction is adjusted with observatory-specific correction curves. But even in this field absolutely flux-calibrated spectra can be found rather rarely [30].

8.7 The Intensity Normalisation of Non Flux Calibrated Spectra
In such cases, the profiles are usually rectified, ie the continuum runs horizontally. This level will now be normalised to \(I_C = 1\) and the wavelength axis of the profile is set to 0. This procedure is mandatory if the equivalent width EW of individual spectral lines must be determined (sect. 7.3) or a long-term intensity monitoring is accomplished.
9 Visible Effects of Quantum Mechanics

9.1 Textbook Example Hydrogen Atom and Balmer Series

The following energy level diagram shows for the simplest possible example, the hydrogen atom, the fixed grid of the energy levels (or "terms") \( n \), which a single electron can occupy in its orbit around the atomic nucleus. They are identical with the shells of the famous Bohr’s atomic model and are also called principal quantum numbers. Which level the electron currently occupies depends on its state of excitation. A stay between the orbits is extremely unlikely. The lowest level is \( n = 1 \). It is closest to the nucleus and also called the ground state.

With increasing \( n \) number (here from bottom to top):

- increases the distance to the nucleus
- increases the total energy difference, in relation to \( n = 1 \)
- the distances between the levels and thus the required energy values to reach the next higher level, are getting smaller and smaller, and finally tend to zero on the Level \( E = 0 \text{ eV} \) (or \( n = \infty \)).

The energy level \( E \) on the level \( n = \infty \) is physically defined as \( E = 0 \text{ eV} \) [5] and also called Ionisation Limit. The level number \( n = \infty \) is to consider as "theoretical", as a limited number of about 200 is expected, which a hydrogen atom in the interstellar space can really occupy [6]. By definition, with decreasing \( n \) number the energy becomes increasingly negative. Above \( E = 0 \text{ eV} \), ie outside of the atom, it becomes positive.

Absorption occurs only when the atom is hit by a photon whose energy matches exactly to a level difference by which the electron is then briefly raised at the higher level (resonance absorption).

Emission occurs when the electron falls back to a lower level and though a photon is emitted, which corresponds exactly to the energy level difference.
**Ionisation** of an atom occurs when the excitation energy is high enough so the negative electron is lifted over the level $E = 0 \ eV$ and leaves the atom. This can be triggered by a high-energy photon (photo ionisation), by heating (thermal ionisation) or a collision with external electrons or ions (collision ionisation).

**Recombination** occurs if an ionised atom recaptures a free electron from the surrounding area and becomes "neutral" again.

### 9.2 The Balmer Series

A group of electron transitions between a fixed energy level and all higher levels is called "Transition Series". For amateurs primarily the Balmer series (red arrow group) is important, because only their spectral lines are in the visible range of the spectrum. It contains the famous H-lines and includes all the electron transitions, which start upward from the second-lowest energy level $n = 2$ (absorption) or end here, "falling" down from an upper level (emission). The Balmer series was discovered and described by the Swiss mathematician and architect (!) Johann Jakob Balmer (1825-1898). The lines of the adjacent Paschen series lie in the infrared, those of the Lyman series in the ultraviolet range.

This sounds very theoretical, but has high practical relevance and can virtually be made "visible" using even the simplest slitless spectrograph! For this purpose the easiest way is to record the classical beginner object, a stellar spectrum of the class A (sect. 13.4). Most suitable is Sirius (A1) or Vega (A0). These stars have a surface temperature of about 10,000 K, which is best suited to generate impressively strong H-Balmer absorptions. The reason for this: Due to thermal excitation at this temperature, the portion of electrons reaches the maximum which occupy already the basic $n = 2$ level of the Balmer series. With further increasing temperature, this portion decreases again, because it is shifted to even higher levels (Paschen series) and will finally be completely released, which results in the ionisation of the H atoms.

In the following Sirius spectrum six of the H-Balmer lines appear, labelled with the relevant Greek letters, starting with Hα in the red region of the spectrum, which is generated by the lowest transition $n_2 - n_3$. From Hε upwards often the respective level number $n$ is used, eg Hζ = H8. Here is nice to see how the line spacing in the Blue area gets more and more closer – a direct reflection of the decreasing amounts of energy, which are required to reach the next higher level. In my feeling this is the aesthetically most pleasing, which the spectroscopy has optically to offer!

---

**Triggering electron transitions**

<table>
<thead>
<tr>
<th>$n_2 - n_3$</th>
<th>$n_2 - n_4$</th>
<th>$n_2 - n_5$</th>
<th>$n_2 - n_6$</th>
<th>$n_2 - n_7$</th>
<th>$n_2 - n_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hζ</td>
<td>Hε</td>
<td>Hγ</td>
<td>Hδ</td>
<td>Hβ</td>
<td>Hα</td>
</tr>
</tbody>
</table>

**Denotation of the lines**
9.3 Spectral Lines of Other Atoms

For all other atoms, i.e. with more than one electron, the description of line formation can be very complex. All atoms have different energy levels and are therefore distinguished by different wavelengths of the spectral lines. Another factor is the number of valence electrons on the outer shell, or how many of the inner levels are already full. Further the main levels are subdivided in a large number of so-called Sublevels and Sub-Sublevels, each of them with completely other implications in respect of quantum mechanics. The energy differences between such sublevels must logically be very low. This explains why metals often appear in dense groups, a few with distances even <1Å! Typical examples are the sodium lines at 5896 Å and 5890 Å, and the famous Magnesium Triplet at 5184, 5173, 5169Å in the solar spectrum. By the hydrogen atoms these sublevels play no practical role, particularly for amateurs, because they are degenerated here [5]. The stay of the electrons within this complex level system is also subject to a set of rules. The best known is probably the so called Pauli Exclusion Principle which demands that the various sublevels may be occupied by only one electron at the same time.

For most spectroscopic amateur activities, detailed knowledge of this complex matter is not really necessary. For those, interested in quantum mechanics, I recommend as “first reading” [5], which provides a good overview on this matter and is relatively easy and understandable to read.
10 Wavelength and Energy

In the above wavelength-calibrated Sirius spectrum we want to calculate the corresponding photon energies $E$ of the hydrogen lines. This is possible with the familiar and simple equation of Max Planck (1858 – 1947):

$$E = h \cdot \nu \quad \{7\}$$

**E** Radiation Energy in Joule [J]

$h$ Planck’s Quantum of Action $[6.626 \cdot 10^{-34} \text{ J s}]$

$\nu$ (Greek: $\nu$) Radiation frequency [s$^{-1}$] of the spectral line.

The Radiation frequency $\nu$ of the spectral line is simply related to the wavelength $\lambda$ [m] by:

$$\nu = \frac{c}{\lambda} \quad \{8\}$$

($c = $ light speed $3 \cdot 10^8 \text{ m/s}$):

Insert {8} into {7}:

$$E = \frac{h \cdot c}{\lambda} \quad \{9\}$$

The most important statement of formulas {7} and {9}: The radiation energy $E$ is proportional to the radiation frequency $\nu$ and inversely proportional to the wavelength $\lambda$.

To express such extremely low amounts of energy in Joules [J] is very impractical and not easy to interpret. Joule has been defined for use in traditional mechanics. In quantum mechanics and therefore also in spectroscopy, the units of electron volts [eV] is in use [5].

$$1\text{ eV} = 1.602 \cdot 10^{-19} \ J \quad \{10\}$$

Further in the optical spectral domain also the wavelengths are extremely small and therefore in astronomy usually measured in angstroms [Å] or nanometer [nm]. One should be aware that 1 Å corresponds about to the diameter of an atom, including its electron shells! In the infrared range also [µm] is in use:

$$1\text{ Å} = 10^{-10} \text{ m}, \quad 1\text{ nm} = 10\text{ Å}, \quad 1\text{ µm (micron)} = 1,000\text{ nm} = 10,000\text{ Å} \quad \{11\}$$

The following simple formulas, suitable for pocket calculators, allow to convert the wavelength $\lambda$ [Å] into energy $E$ [eV] and vice versa:

$$\lambda[\text{Å}] = \frac{12403}{E[\text{eV}]} \quad \{12\}$$

$$E[\text{eV}] = \frac{12403}{\lambda[\text{Å}]} \quad \{13\}$$
To the wavelengths $\lambda$ of the hydrogen lines we can now calculate with formula {13} the corresponding values of the photon energy $E_p$.

In professional publications, spectra in the UV region are often calibrated in photon energy $E_p$ [eV], instead of the wavelength $\lambda$.

On the left side, the so-called Balmer edge or Balmer jump is marked with a red bar. The Balmer series ends here at 3647 Å and the continuum suffers a dramatic drop in intensity, due to the huge phalanx of highly concentrated absorption lines. The $E_p$ Value of a spectral line corresponds to the energy difference between the initial and final level of the causal electron transition. It fits therefore also to the arrow lengths in the following energy level diagram. In the spectrum above e.g. 2.55eV corresponds to the transition $n_2 - n_4$ or H$\beta$.

This relationship enables now to calculate the energy levels $E$ of the H-Balmer series. Since by definition the Energy on the level $n = \infty$ is set to $E = 0$ we shift the $E_p$ value of the Balmer edge 3.40 eV with negative sign to the initial level of the Balmer series $n = 2$. 3.40 eV corresponds also to the energy difference between the initial level $n_2$ and the Balmer edge, as well as to the minimum required energy to ionise a hydrogen atom from the initial level $n = 2$.

For the calculation of the other energy levels the $E_p$ values of the corresponding level differences must now be subtracted from 3.40 eV. For the energy level eg $n = 3$, the subtraction $3.40 \text{ eV} - 1.89 \text{ eV} = 1.51 \text{ eV}$ is required (1.89 eV = $E_{\text{pia}}$). Per definition the values of the energy levels have here negative signs.
11 Ionisation Stage and Degree of Ionisation

In the diagram above the energy for the lowest excitation level of the Lyman Series \( n = 1 \) is \( E = -13.6 \, \text{eV} \). Converted with formula \( 12 \), this gives the well-known Lyman limit or Lyman edge in the UV range with wavelength \( \lambda = 912 \, \text{Å} \). It is functionally equivalent to the “Balmer edge” of the Balmer Series. This value is very important for astrophysics, because it defines the minimum required energy to ionise the H-atom from its ground state \( n = 1 \). This level is only achievable by very hot stars of the O- and early B-Class. [3]. The very high UV radiation of such stars ionises first the hydrogen clouds which are shining due to emitted photons by the subsequent recombination (H II regions, eg M42, Orion Nebula, sect. 21).

The term “Ionisation stage” refers here to the number of electrons, which an ionised atom has lost to the space (Si IV, Fe II, H II, etc.). This must not be confused with the term “Degree of ionisation” in plasma physics. It defines for a gas mixture the ratio of atoms (of a certain element) that are ionised into charged particles, regarding the temperature, density and the required ionisation energy of the according element. This “Degree” is determined in astrophysics with the famous Saha equation.

Unfortunately, in astrophysics the chemical form of notation is not in use but instead of it another somewhat misleading version. The neutral hydrogen is denoted by chemists with \( H \), and the ionised with \( H^+ \), which is clear and unambiguous. On the other hand astrophysicists, denote already the neutral hydrogen with an additional Roman numeral as \( H I \) and the ionised with \( H II \). The doubly ionised calcium is referred by chemists with \( Ca^{++} \), for astrophysicists this corresponds to \( Ca III \). Si IV is for example triply ionised silicon \( Si^{+++} \). This system therefore works according to the \( (n-1) \) principle, ie in astrophysics the ionisation stage of an atom is always by 1 lower than the Roman numeral. A high ionisation stage of atoms always means that very high temperatures must be involved in the process.
12 Forbidden Lines or –Transitions

Based on the already presented theories this phenomenon can only roughly be explained. For a more comprehensive understanding, further quantum mechanical knowledge would be needed. For practical spectroscopy, this is not really necessary.

Most amateurs have an [O III] filter for the contrast enhancement of emission nebulae. It lets pass the two green emission lines of the doubly ionised oxygen [O III] at 4959Å und 5007Å. These lines are generated here by so-called "Forbidden Transitions" between the energy levels (sect. 21). The initial levels of such transitions are called “metastable”, because they are highly sensitive to impacts and an electron must remain here for a quite long time (several seconds to minutes) until it performs the forbidden “jump”. These circumstances increase drastically the likelihood that this state is destroyed before the transition happens.

"Forbidden" therefore means that in dense gases, such as on the earth's surface or in stellar atmospheres, these transitions are extremely unlikely, because they are prevented by collisions with other particles. This disturbing effect occurs very rarely within the extremely thin gases of the interstellar space. Thus such transitions are possible here. Such forbidden lines are denoted within square brackets eg [O III], [N II], [Fe XIV].

In addition to nitrogen bands, the forbidden airglow line [OI] (5577.35Å) is the main cause for the formation of the usually greenish polar lights in the extremely thin upper layers of the atmosphere. For the generation of higher ionisation stages such as [O III] the required ionisation energy is missing here.
13 The Spectral Classes

13.1 Preliminary Remarks

Basic knowledge of the spectral classes is an indispensable prerequisite for a reasonable spectroscopic activity. Combined with appropriate skills and tools, this classification system contains a considerable qualitative and quantitative information potential about the classified objects. The average-equipped amateur will hardly ever come into the embarrassment, that he really must determine an unknown classification of a star, unless of certainly recommendable didactic reasons. The spectral classes can nowadays be obtained from Internet sources [100], planetarium programs, etc. For a deeper understanding of the classification system, a rough knowledge of the historical development is very useful since from each development step something important remained until to date!

13.2 The Fraunhofer Lines

At the beginning of the 19th Century, the physicist and optician Joseph von Fraunhofer (1787-1826) investigated, based on the discovery of Wollaston, the sunlight with his home-built prism spectroscope. He discovered over 500 absorption lines in this very complex spectrum. The more intensive of them he denoted with the letters A – K, at that time still unaware of the physical context.

Picture below: Original drawing by Fraunhofer from Internet sources. This line names can frequently be found even in recent papers!

Fraunhofer has been studied the brighter stars with this spectroscope, and already recognised that in the spectrum of Sirius broad strong lines dominate and Pollux shows a similar pattern as the spectrum of the sun! Further he observed the spectrum of Betelgeuse which shows barely discrete absorption lines but broad absorption bands.

The table on the right and the graphics below show how this system was expanded later on. (Source: NASA).
Here are some of the identified Fraunhofer lines in the solar spectrum, obtained with the DADOS Spectrograph in the range of 4200 – 6700 Å (200L/mm).

Below left, the H and K lines of ionised calcium. Ca II are the most intense of the sun-grown absorption lines (3968/3934 Å). Below right is the beautiful so-called A-band (7594 – 7621 Å), caused by the O$_2$ molecule in the earth’s atmosphere (DADOS and 900L/mm grating).

13.3 Further Development Steps

In the further course of the 19th to the early 20th Century astronomy, and particularly the astro-spectroscopy benefited from impressive advances in chemistry and physics. So it became increasingly possible to assign the spectral lines to chemical elements - primarily the merit of Robert Bunsen (1811 - 1899) and Gustav Kirchhoff (1824 - 1887)

Decisively, Father Angelo Secchi (1818 - 1878) from the Vatican Observatory influenced the future path of the stellar spectral classification. He is therefore referred by many sources as the father of the modern astrophysics. He subdivided the stellar spectra into five groups according to specific characteristics (I-V “Secchi Classes”).

**Type I** Bluish-white stars with relatively simple spectra, which are dominated by small but very bold lines. These are distributed like thick rungs over the spectral stripe and turned out later as the hydrogen lines of the famous Balmer series. This simple characteristic allows even beginners, to roughly classify such stars into the currently used A- or late B-Class (Sirius, Vega, Castor).

**Type II** Yellow shining stars with complex spectra, dominated by numerous metal lines, like they exhibit the Sun, Capella, Arcturus, Pollux (now Classes ≈ F, G, K).
**Type III** Reddish-orange stars with complex band spectra and a few discrete lines. The absorption bands get darker (more intense) to the shortwave (blue) side. Such features show, e.g. Betelgeuse, Antares, and Mira. Not until 1904, it became clear that these absorption bands are mainly caused by the titanium oxide molecule TiO (today's Class M).

**Type IV** Very rare reddish stars with absorptions bands getting darker (more intense) to the longwave (red) side. Angelo Secchi already recognised that it is generated by carbon (sect. 5.4)!

**Type V** Finally, stars with "bright lines" – emission lines as we know today.

### 13.4 The Harvard System

It soon became clear that the classification system of Secchi was too rudimentary. Based on a large number of spectra and preliminary work by Henry Draper, Edward Pickering (1846-1919) refined Secchi's system by capital letters from A – Q. The letter A corresponded to the Secchi class type I for stars with dominant hydrogen lines. Finally this classification solely survived up to the present time! As director of the Harvard Observatory, he employed many women, for its time a truly avant-garde attitude. Three women of his staff took care of the classification problem until after many detours and “meanders” the system of Annie J. Cannon (1863 - 1941) became widely accepted around the end of the World War I.

Its basic structure has survived until today and is essentially based on the letters O, B, A, F, G, K, M. The well-known and certainly later created mnemonic: Oh Be A Fine Girl Kiss Me. With this system, even today still over 99% of the stars can be classified. This sequence of letters follows the decreasing atmospheric temperature of the classified stars, starting from the very hot O-types with several 10,000 K up to the cool M-types with about 2,400 –3,500 K. This reflects the absolutely ground-breaking recognition that the spectra depend mainly on the atmospheric temperature of the star and secondarily only on other parameters such as chemical composition, density, rotation speed etc. This may not be really surprising from today's perspective, since the shares of hydrogen and helium with 75% and 24%, even about 13.7 billion years after the “Big Bang” still comprise about 99% of the elements in the universe. This systematic also forms the horizontal axis of the almost simultaneously developed Hertzsprung Russell Diagram (sect. 14). It was later complemented by the classes:

- **R** for Cyan (CN) and Carbon Monoxide (CO)
- **N** for Carbon
- **S** for very rare stars, whose absorption bands are generated, instead of TiO, by zirconium oxide (ZrO), yttrium oxide (YO) or lanthanum oxide (LaO). Moreover, the entire class system was further subdivided with additional decimal numbers from 0–10. Examples: Sun G2, K0 Pollux, Vega A0, Sirius A2, Procyon F5.
13.5 “Early” and “Late” Spectral Types

One of the faulty hypotheses on the long road to this classification system postulated that the spectral sequence from O to M represents the chronological stages of a star. Unaware at that time of nuclear fusion as a "sustainable" energy source, also the hypothesis was discussed, that the stars could generate their energy only by contraction, ie starting very hot and finally ending cool. This error has subsequently influenced the terminology until today. Thus the O, B, A classes are called "early", the F and G classes "medium" and K and M as "late" types. This systematic is also applied within a class. So MO is called an "early" and M8 a “late” M-type. Logically is for example M1 "earlier" than M7.

13.6 The MK (Morgan Keenan) or Yerkes System

Later on, the progress in nuclear physics and the increasing knowledge of the stellar evolution required a further adaptation and extension of the system. It was eg recognised, that within the same spectral class, stars can show totally different absolute luminosities, mainly caused by different stages of stellar development.1943, as another milestone, the classification system was extended with an additional Roman numeral by Morgan, Keenan and Kellmann from the Mt Wilson Observatory. This second dimension of the classification specifies the so called six luminosity classes.

<table>
<thead>
<tr>
<th>Luminosity class</th>
<th>Star type</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Luminous Super Giants</td>
</tr>
<tr>
<td>Ia-0, Ia, lab, Ib</td>
<td>Subdivision of the Super Giants according to decreasing luminosity</td>
</tr>
<tr>
<td>II</td>
<td>Bright Giants</td>
</tr>
<tr>
<td>III</td>
<td>Normal Giants</td>
</tr>
<tr>
<td>IV</td>
<td>Sub Giants</td>
</tr>
<tr>
<td>V</td>
<td>Dwarfs or Main Sequence Stars</td>
</tr>
<tr>
<td>VI</td>
<td>Sub Dwarfs (rarely used, as specified by prefix)</td>
</tr>
<tr>
<td>VII</td>
<td>White Dwarfs (rarely used, as specified by prefix)</td>
</tr>
</tbody>
</table>

This system classifies the Sun as a G2V star, an ordinary Dwarf on the main sequence of the Hertzsprung Russel Diagram. Sirius, classified as A1Vm, is also a Dwarf on the main sequence. Betelgeuse, as a Super Giant and rated with M1–2 Ia–Iab, moves as a variable between M1 and M2 and fluctuates between the luminosity class Ia and lab. It has given up the Dwarf stage on the main sequence long ago and expanded into a Super Giant.

13.7 Further Adaptations up to the Present

Up to the presence, this classification system has been adapted to the constantly growing knowledge. Thus, new classifications for rare, stellar "exotics" have emerged, which today even amateurs successfully deal with. Furthermore with additional lower-case letters attached as a prefix or suffix, unusual phenomena, such as a higher than average metal content or "metallicity" are referred. Some of these supplements are, however, over determining, eg because White Dwarfs, Sub Dwarfs and Giants are already specified by the luminos-
ity class. Caveat: Dwarf or main sequence stars must not be confused with the White Dwarfs. The latter are extremely dense, burned-out "stellar corpses" (sect. 14.3).

Examples: Sirius A: $A1V\!m$, metal rich main sequence star (Dwarf), spectral class $A1$
Sirius B: $DA2$, White Dwarf (or „Degenerate”) spectral class $A2$
Omicron Andromedae: $B6IIle$, Omicron Ceti (Mira): $M7IIle$
Kapteyn’s star: $sdM1V$

<table>
<thead>
<tr>
<th>Suffix</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>Sharp lines</td>
</tr>
<tr>
<td>c</td>
<td>Extraordinary sharp lines</td>
</tr>
<tr>
<td>B</td>
<td>Broad lines</td>
</tr>
<tr>
<td>A</td>
<td>Normal lines</td>
</tr>
<tr>
<td>comp</td>
<td>Composite spectrum</td>
</tr>
<tr>
<td>e</td>
<td>H- emission lines by B- and O Stars</td>
</tr>
<tr>
<td>F</td>
<td>He- and N- emission lines by O-stars</td>
</tr>
<tr>
<td>Em</td>
<td>Metallic emission lines</td>
</tr>
<tr>
<td>K</td>
<td>Interstellar absorption lines</td>
</tr>
<tr>
<td>M</td>
<td>Strong metal lines</td>
</tr>
<tr>
<td>n / nn</td>
<td>Diffuse lines/strongly diffuse lines</td>
</tr>
<tr>
<td>Wk</td>
<td>Weak lines</td>
</tr>
<tr>
<td>p, pec</td>
<td>Peculiar spectrum</td>
</tr>
<tr>
<td>Sh</td>
<td>Shell</td>
</tr>
<tr>
<td>V</td>
<td>Variation in spectrum</td>
</tr>
<tr>
<td>Fe, Mg...</td>
<td>Excess or deficiency (−) of the spec. element</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Prefixes</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>Dwarf</td>
</tr>
<tr>
<td>sd</td>
<td>Sub Dwarf</td>
</tr>
<tr>
<td>g</td>
<td>Giant</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Special classes</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>Nova</td>
</tr>
<tr>
<td>P</td>
<td>Planetary Nebulae</td>
</tr>
<tr>
<td>D</td>
<td>Dwarf, +additional letter for $O, B, A$ spectral class</td>
</tr>
<tr>
<td>W</td>
<td>Wolf-Rayet Star + additional letter for $C, N$, or $O$-lines</td>
</tr>
<tr>
<td>S</td>
<td>Stars with zirconium oxide absorption bands</td>
</tr>
<tr>
<td>C</td>
<td>Carbon stars</td>
</tr>
<tr>
<td>L, T,</td>
<td>Brown dwarfs</td>
</tr>
<tr>
<td>Y</td>
<td>Theoretical class for brown dwarfs &lt;600 K</td>
</tr>
</tbody>
</table>

Nowadays the special classes $P$ (Planetary Nebulae) and $Q$ (Novae) are barely in use!

The suffixes are not always applied consistently. We often see other versions. In the case of shell stars e.g. $pe$, or $shell$ is in use.
13.8 The Rough Determination of the Spectral Class

The rough, one-dimensional determination of the spectral classes \( O, B, A, F, G, K, M \), is easy and even feasible for slightly advanced amateurs. The distinctive criteria are striking features such as line- or band spectra, as well as absorption- or emission lines, which appear prominently in certain spectral classes and vice versa are very weak or even completely absent in others. But the determination of the decimal subclasses and even more, the additional determination of the luminosity class (second dimension), require well-resolved spectra with a large number of identified lines, as well as deeper theoretical knowledge. Possible distinctive criterions are eg the intensity ratio or the FWHM values of certain spectral lines. On this topic exists an extensive literature eg [4]. Here follows just a brief introduction into the rough, one dimensional determination of the spectral class. A further support is the *Spectroscopic Atlas for Amateur Astronomers* [33]. The following figure shows superimposed and lowly-resolved the entire sequence of the spectral classes \( O - M \), as it can be found in [33].
What is already clearly noticeable here?

- In the upper third of the table (B2-A5), the strong lines of the H-Balmer series, i.e. $H\alpha$, $H\beta$, $H\gamma$, etc. They appear most pronounced in the class A2 and are weakening from here towards earlier and later spectral classes.

- In the *lower quarter* of the table (K5-M5) the eye-catching shaded bands of molecular absorption spectra, mainly due to titanium oxide ($TiO$).

- Just underneath the half of the table some spectra (F5-K0), showing only few prominent features, but charged with a large number of fine metal lines. Striking features here are only the Na I double line (Fraunhofer D1, 2) and in the “blue” part the impressive Fraunhofer lines of Ca II (K + H), gaining strength towards later spectral classes. *Fraunhofer H* at $\lambda$ 3968 starts around the *early F-class* to *overprint* the weakening H$\varepsilon$ hydrogen line at $\lambda$ 3970. In addition, the H-Balmer series is further weakening towards later classes.

- Finally on the top of the table the extremely hot O-class with very few fine lines, mostly ionised helium (He II) and multiply ionised metals. The H-Balmer series appears here quite weak, as a result of the extremely high temperatures. The telluric $H_2O$ and $O_2$ absorption bands are reaching high intensities here, because the strongest radiation of the star takes place in the ultraviolet whereas the telluric absorption bands are located in the undisturbed domain near the infrared part of the spectrum. By contrast the maximum radiation of the *late* spectral classes takes place in the infrared part, enabling the stellar TiO absorption bands to overprint here the telluric lines.

- In the spectra of hot stars (~ classes from early A – O) the double line of *neutral* sodium Na I (Fraunhofer D$_{1,2}$) must imperatively be of interstellar origin. Neutral sodium Na I has a very low ionisation energy of just 5.1 eV and can therefore exist only in the atmospheres of relatively cool stars. The wavelengths of the ionised Na II lie already in the ultraviolet range and are therefore not detectable by amateur equipment.
Here follow two different flow diagrams, for the rough one-dimensional determination of the spectral class: Sources: Lectures from University of Freiburg i.B. [56] and University of Jena.

**Version 1**

Applied lines:
- Fraunhofer K (Ca II K 3934Å)
- Fraunhofer H (Ca II H 3968Å)
- Fraunhofer G Band (CH molecular 4300–4310Å)
- Balmer line Hγ, 4340Å
- Mangan Mn 4031/4036Å
- Titanoxide Bandhead: TiO 5168Å

The intensity I of the compared lines (e.g., Ca II K/Hγ) is calculated by the Peak intensities (sect. 7.1).

\[ P = \frac{I}{I_c} \quad \{14\} \]

**Version 2**

Applied lines:
- Fraunhofer K (Ca II K 3934Å)
- Fraunhofer H (Ca II H 3968Å)
- Balmer line Hγ, 4340Å
- Fe I, 4325Å

Balmer lines visible?

- yes
  - H and K visible?
    - yes
      - Spectral type O
    - no
      - Spectral type K, M
  - no
    - Spectral type B

H and K visible?

- yes
  - K/Hγ<1?
    - yes
      - Spectral type A0-A5
    - no
      - K/Hγ=1?
        - yes
          - Spectral type A5
        - no
          - Hy/Fe I<1? (4325)

- no
  - Spectral type later A5

Spectral type A5

- no
  - Hy/Fe I=1? (4325)
    - yes
      - Spectral type later G5
    - no
      - Spectral type later G6

Spectral type A6-F

G-band/TiO 5168

- yes
  - Mn 4031, 4036
    - yes
      - Spectral type A6-F
    - no
      - Spectral type later A5

F3–F9

- yes
  - Mn 4031, 4036
    - yes
      - Spectral type A6-F
    - no
      - Spectral type later A5

F0–F3

- yes
  - Mn 4031, 4036
    - yes
      - Spectral type A6-F
    - no
      - Spectral type later A5
Here some additional classification criteria (see also [3], [4], [33] and [80]).

**Course of the Continuum curve:**

Already by comparing of non-normalised raw spectra, eg an early B- star against a late K-star, one can observe that the intensity maximum of the pseudo continuum is significantly blue-shifted (Wien's Displacement Law). Extremely hot O- and early B- stars radiate mainly in the ultraviolet (UV), cold M- stars chiefly in the infrared range (IR).

**O-Class:** singly ionised helium He II, also as emission line. Neutral He I, doubly ionised C III, N III, O III, triply ionised Si IV. H- Balmer series only very weak. Maximum intensity of the continuum is in the UV range.

Examples: *Alnitak (Zeta Orionis): O9 Ib, Mintaka (Delta Orionis): O9.5 II,

**B- Class:** Neutral He I in absorption strongest at B2, Fraunhofer K-Line of Ca II becomes faintly visible, further singly ionised OII, Si II, Mg II. H- Balmer series becomes stronger.

Examples: *Spica: B1 III-IV, Regulus (Alpha Andromedae): B8 IVp Mn Hg Algol (Beta Persei): B8V, as well as all bright stars of the Pleiades.

**A-Class:** H- Balmer lines are strongest at A2, Fraunhofer H+K lines Ca II become stronger, neutral metal lines become visible, helium lines (He I) disappear.

Examples: *Wega: A0 V, Sirius: A1 V m Castor A2 V m, Deneb: A2 1a Denebola: A3 V, Altair: A7V,

**F-Class:** H- Balmer lines become weaker, H+K lines Ca II, neutral and singly ionised metal lines become stronger (Fe I, Fe II, Cr II, Ti II). The striking “line double” of G-Band (CH molecular) and Hγ line can only be seen here and forms the unmistakable "Brand" of the middle F-class [33]!

Examples: *Caph (Beta Cassiopeiae): F2III-IV, Mirphak (Alpha Persei): F5 Ib, Polaris: F7 Ib-II, Sadr (Gamma Cygni): F8 Ib, Procyon: F5 IV-V

**G-Class:** Fraunhofer H+K lines Ca II very strong, H- Balmer lines get further weaker, Fraunhofer G- Band becomes stronger as well as many neutral metal lines eg Fe I, Fraunhofer D-line (Na I).

Examples: *Sun: G2V, the brighter component of Alpha Centauri G2V, Mufrid (Eta Bootis): G0 IV, Capella G5IIIe + G0III (binary star composite spectrum).

**K-Class:** Is dominated by metal lines, H- Balmer lines get very weak, Fraunhofer H+K Ca II are still strong, Ca I becomes strong now as well as the molecular lines CH-CN. By the late K- types first appearance of TiO bands.

Examples: *Pollux: K0IIIb, Arcturus: K1.5 III Fe, Hamal (Alpha Arietis): K2 III Ca, Aldebaran: K5 III

**M-Class:** Molecular TiO- bands get increasingly dominant, many strong neutral metal lines, eg Ca I. Maximum of the continuum is in the IR range.

Examples: *Mirach (Beta Andromedae): M0 Illa, Betelgeuse: M1-2 Ia-Iab, Antares: M1.5 Iab-b, Menkar (Alpha Ceti): M 1.5 Illa, Scheat, (Beta Pegasi): M3 III Tejat Posterior (mű Gemini): M3 III Ras Algheti: (Alpha Herculis): M5III,
The following chart shows the change in the line intensity (equivalent width EW) of characteristic spectral lines as a function of the spectral type or the temperature. It was developed 1925 in a dissertation by Cecilia Payne Gaposhkin (1900 – 1979).

This chart is not only of great value for determining the spectral class, but also prevents by the line identification from large interpretation errors. Thus becomes immediately clear that the photosphere of the Sun (spectral type G2V) is a few thousand degrees too cold to show helium He I in a normal (photospheric) solar spectrum. He I is visible only during solar eclipses as an emission line in the so called flash spectrum, which is produced mainly in the much hotter solar chromosphere.

13.9 Effect of the Luminosity Class on the Line Width

Here we see within the same spectral class how the line width increases with decreasing luminosity. This happens primarily due to the so-called "pressure broadening", ie the broadening of spectral lines due to increasing gas pressure. The main reason is the increasing density of the stellar atmosphere with decreasing luminosity, ie the star becomes smaller, less luminous and denser.

Most densely are the atmospheres of the white dwarfs, class VII, least densely among the Super Giants of class I. This effect is strongest by the H-Balmer series of class A (below left). Already by the F-class (same wavelength domain, below right), this effect is barely noticeable. This trend continues towards the later spectral classes (excerpts from [33]).
14 The Hertzsprung - Russell Diagram (HRD)

14.1 Introduction to the Basic Version

The HRD was developed in 1913 by Henry Russell, based on the work by Ejnar Hertzsprung. It is probably the most fundamental and powerful illustrating tool in astrophysics. On the topic of stellar evolution and HRD an extensive literature exists, which must be studied anyway by the ambitious amateur. Here it is illustrated, how a wealth of information can be gained about a star just with the help of the spectral class and the HRD. The following figure shows the basic version of the HRD with the luminosity (compared to the sun), plotted against the spectral type. The luminosity classes Ia – VII are marked by lines within the diagram. Further visible is the Main Sequence, identical to the line for luminosity class V, and the two branches, where the Red Giants and White Dwarfs are gathering.

The spectral class unambiguously determines the position of a star inside the diagram and vice versa. The position of the sun, with the classification G2V is already marked (yellow disk). With this diagram the luminosity of the spectral class, in comparison to the sun, can already be determined – in the case of the sun $10^0 = 1$.

In the following it will be demonstrated, how further parameters of the star can be determined, just by modification of the HRD-scales.
14.2 The Absolute Magnitude and Photospheric Temperature of the Star

The following version of the HRD shows on the horizontal axis at the upper edge of the diagram, the corresponding temperature of the stellar atmosphere, ie the photosphere of the star, where the visible light is produced. The position of the Sun (G2V) is here also marked with a yellow disc. At the top of the chart the temperature of about 5,500 ° K can be read, on the left the absolute brightness (Absolute Magnitude) with ca. 4M5. This corresponds to the apparent brightness, which a star generates in a normalised distance of 10 parsecs, or some 32.6 light-years.

Redrawn and supplemented following a graphic from: www.bdaugherty.tripod.com

The chart is further populated with a variety of common stars. The sun, along with Sirius, Vega, Regulus, Spica, etc. is still a Dwarf star on the Main Sequence. Arcturus, Aldebaran, Capella, Pollux, etc. have already left the main sequence and shining now on the Giant Branch with the Luminosity Class III, Betelgeuse, Polaris, Rigel, Deneb, in the range of the giants, with their respective classes Ia-Ib. On the branch of the White Dwarfs we see the companion stars of Sirius and Procyon.
14.3 The Evolution of the Sun in the HRD

The following simplified short description is based on [1]. It demonstrates how the spectral class allows the determination of the stellar state of development. In the diagram below, the evolutionary path of the sun is shown. By the contraction of a gas and dust cloud, at first a protostar is formed, which subsequently moves within some million years on the Main Sequence. Here, it stabilises at first the luminosity by about 70% of the today’s value. Within the next 9-10 billion years the luminosity increases to over 180%, whereby the spectral class G2, e.g. the photospheric temperature, remains more or less constant [1]. During this period as a Dwarf- or Main Sequence star, hydrogen is fused into helium.

Towards the end of this stage the hydrogen is burning in a growing shell around the helium core. The sun becomes now unstable and expands to a Red Giant of the M-class (luminosity class ca. II). It moves now on the Red Giant Branch (RGB) to the top right of the HRD, where after 12 billion years it comes to the ignition of the helium nucleus, the so called Helium Flash. The photosphere of the Red Giant expands now almost to the Earth’s orbit. By this huge expansion the gravity acceleration within the stellar photosphere is reduced dramatically and the star loses therefore at this stage about 30% of its mass [1].

Subsequently the giant moves with merging helium core on the Horizontal Branch (HB) to the intermediate stage of a Yellow Giant of the K-class (lasting about 110M years) and then via the reverse loop of the Asymptotic Giant Branch (AGB) to the upper left edge of the HRD (details see [33]). The experts seem still be divided whether the sun is big enough to push off at this stage the remaining shell as a visible Planetary Nebula [1], [2]. Assured however is the final shrinkage process to an extremely dense White Dwarf of about Earth’s size. After further cooling, down on the branch of the White Dwarfs, the sun will finally get invisible as a Black Dwarf, and disappear from the HRD. During its live the Sun will pass through a large part of the spectral classes, but with very different luminosities.
14.4 The Evolution of Massive Stars

In the next subsection it will be shown how the length of stay on the main sequence is dramatically reduced with increasing stellar mass. Highly complex nuclear processes cause complex pendulum like movements in the upper part of the HRD, showing various stages of variables (more details see [33]). During this period also many of the heavy elements in the periodic table are generated. Massive Stars about >8–10 solar masses, will not end as White Dwarfs, but explode as Supernovae [2]. Depending on the mass of the star it finally remains a Neutron Star, if the rest of the stellar mass is not greater than about 1.5–3 solar masses (TOV limit: Tolman-Oppenheimer-Volkoff). Above this limit, eg stars with initial >15–20 solar masses, it ends in a Black Hole.

14.5 The Relation between Stellar Mass and Life Expectancy

The effect of the initial stellar mass is crucial for its further life. First, it decides which place it occupies on the Main Sequence. The larger the mass the more left in the HRD or "earlier" in respect of the spectral classification. Second, it has a dramatic impact on his entire life expectancy, and the somewhat shorter period of time that he will spend on the Main Sequence. This ranges from roughly some million years for the early O- types up to >100 billion years for Red Dwarf stars of the M class. This is due to the fact that with increasing mass the stars consume their "fuel" over proportionally faster. This relationship becomes evident in the following table for Dwarf Stars on the Main Sequence (luminosity class V), together with other parameters of interest. The values are taken from [53]. Mass, radius and luminosity are given in relation to the values of the Sun (\(\odot\)).

<table>
<thead>
<tr>
<th>Spectral class, Main Sequence</th>
<th>Mass (M/M_\odot)</th>
<th>Stay on Main Sequence [Y]</th>
<th>Temperature of stellar atmosphere</th>
<th>Radius (R/R_\odot)</th>
<th>Luminosity (L/L_\odot)</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>20–60</td>
<td>10–1M</td>
<td>&gt;25 – 30'000 K</td>
<td>9-15</td>
<td>90'000 –800'000</td>
</tr>
<tr>
<td>B</td>
<td>3–18</td>
<td>400–10M</td>
<td>10'500–30'000 K</td>
<td>3.0–8.4</td>
<td>95 – 52'000</td>
</tr>
<tr>
<td>A</td>
<td>2–3</td>
<td>3bn–440M</td>
<td>7'500 – 10'000 K</td>
<td>1.7–2.7</td>
<td>8 – 55</td>
</tr>
<tr>
<td>F</td>
<td>1.1–1.6</td>
<td>7bn–3M</td>
<td>6'000 – 7'200 K</td>
<td>1.2–1.6</td>
<td>2.0 – 6.5</td>
</tr>
<tr>
<td>G</td>
<td>0.9–1.05</td>
<td>15bn–8bn</td>
<td>5'500 – 6'000 K</td>
<td>0.85–1.1</td>
<td>0.66 – 1.5</td>
</tr>
<tr>
<td>K</td>
<td>0.6–0.8</td>
<td>&gt;20bn</td>
<td>4'000 – 5'250 K</td>
<td>0.65–0.80</td>
<td>0.10 – 0.42</td>
</tr>
<tr>
<td>M</td>
<td>0.08–0.5</td>
<td>2'600 – 3'850 K</td>
<td>0.17–0.63</td>
<td>0.001 – 0.08</td>
<td></td>
</tr>
</tbody>
</table>

The length of stay of the K- and M-class Dwarf Stars differs considerably depending on the source. It has anyway more theoretical importance, since these stars have a significantly longer life expectancy than the current age of the universe from estimated 13.7bn years!
14.6 Age Determination of Star Clusters

The relationship between the spectral class and the according time, spent on the Main Sequence, allows the age estimation of star clusters – this under the assumption that such clusters are formed within approximately the same period from a gas and dust cloud. If the spectral classes of the cluster stars are transferred to the HRD, it gives the following interesting picture: The older the cluster, the more right in the diagram (i.e., "later") the distribution of stars turns off from the Main Sequence up to the realm of giants and Super Giants (so called Turn off Point). M67 belongs with more than 3 billion years to the oldest open clusters, i.e., the O-, B- and A-, as well as the early F-types have already left the Main Sequence, as shown on the chart. However all the bright stars of the Pleiades (M45), still belong to the middle to late part of the B-class. This cluster must therefore necessarily be younger than M67 (about 100M years). One can also say that the Main Sequence "burns off" with increasing age of the cluster like a candle from top to down.

The horizontal axis of the HRD is divided instead of spectral types with the equivalent values of the Color-Magnitude Diagram (CMD). This photometrically determined B - V color index is the brightness difference of the object spectrum (magnitudes) between the blue range (at 4,400 Å) and the "visual" range at 5,500 Å (green). The difference = 0 corresponds to the spectral class A0 (standard star Vega). Earlier classes O, B have negative values, later classes are positive. For the Sun (G2), this value is + 0.62, for Betelgeuse (M1) +1.85.
15 The Measurement of the Radial Velocity

15.1 The Doppler Effect

The Doppler principle, named after the Austrian physicist Christian Doppler 1803 – 1853, enables the determination of the radial velocity. One of the classic explanation models is the changing pitch, emitted by the siren of a passing emergency vehicle. The same effect can be observed in the entire range of electromagnetic waves, including of course the visual light. Observed from $B$ this effect is caused by the radial velocity component $\mathbf{V}_r$ of a radiation source $S$ (e.g. a star), moving with the velocity $\mathbf{V}$.

If $\mathbf{V}_r$ is directed away from the observer, the observed wavelength appears stretched and the spectrum therefore redshifted. In the opposite case it appears compressed and the spectrum blue shifted.

In the spectrum of $S$, we can measure the wavelength shift $\Delta \lambda$. The radial velocity $\mathbf{V}_r$ can then simply be calculated with the Doppler formula:

$$v_r = \frac{\Delta \lambda}{\lambda_0} \cdot c$$  \hspace{1cm} \text{(15)}

$$\Delta \lambda = \frac{v_r \cdot \lambda_0}{c}$$  \hspace{1cm} \text{(16)}

$\Delta \lambda$ = Measured shift in wavelength of a given spectral line
$\lambda_0$ = Wavelength of the considered spectral line in a stationary system
$c$ = Speed of light 300'000 km/s
- If the spectrum is blue shifted, the object is approaching us and $v_r$ becomes negative
- If the spectrum is red shifted, the object is receding and $v_r$ becomes positive.
15.2 The Measurement of the Doppler Shift

For radial velocity measurements ($v_r$) in most cases, the Doppler shift is determined by the difference between a specific spectral line (e.g. H$\alpha$) and their well-known "nominal wavelength" $\lambda_0$ in a stationary laboratory spectrum. For this purpose with the unchanged spectrograph setup a calibration lamp spectrum is recorded, immediately before and/or after the object spectrum. The calculation of $v_r$ is finally enabled by formula (15). A rudimentary and less accurate alternative to the calibration lamp is to record a spectrum of a star with intensive, well known lines and a very low radial velocity.

15.3 Radial Velocities of nearby Stars

The radial velocities of stars in the vicinity of the solar system reach for the most part only one-or two-digit values in [km/s]. Examples: Aldebaran +54 km/s, Sirius –8.6 km/s, Betelgeuse +21 km/s, Capella +22 km/s [100].

The corresponding shifts $\Delta \lambda$ are therefore very low, usually just a fraction of a 1Å. For $\Delta \lambda = 1$Å and based on the H$\alpha$ line (6563 Å), $v_r$ corresponds to ~46 km/s (formula (15)). Therefore highly-resolved and accurately calibrated spectra are necessary.

15.4 Relative Displacement within a Spectrum caused by the Doppler Effect

Textbook examples for this effect are the so-called P Cygni profiles (sect. 5.5). For the determination of the expansion velocity of the stellar envelope neither an absolutely wavelength calibrated spectrum nor a heliocentric correction [30] is required. The measurement of the relative displacement between the absorption and the emission part of the P Cygni profile is sufficient. For P Cygni this displacement within the H$\alpha$ line amounts to some 4.4 Å, corresponding to an expansion velocity of ~200 km/s [33].

15.5 Radial Velocities of Galaxies

Even for the brightest galaxies in the Messier catalogue the recording of the spectra requires large telescope apertures and exposure times of dozens of minutes. In this area the famous cosmological Red Shift by Edwin Hubble (1889-1953, usually depicted with a pipe) has now to take into account by the interpretation of extragalactic spectra. The difficulty here is the distinction between the kinematic Doppler Shift, due to the relative proper motions of the galaxies and the cosmological Red Shift, caused by the intrinsic expansion of the relativistic space-time lattice. The latter phenomenon has nothing to do with the Doppler Effect!

Within the range of the Messier Galaxies, i.e. a radius of about 70M ly, the proper motion of the Galaxies still dominates. Six of the 38 galaxies are moving against the "cosmological trend", i.e. with blue-shifted spectra, towards our Milky Way! These include M31 (Andromeda) with about −300 km/s, and M33 (Triangulum) with some −179 km/s [101]. With increasing distance, however, the cosmological share of the measured Red Shift becomes more and more dominant. From a distance of some 100 mega-parsecs [1 Mpc = 3.26M ly], the influence of the Doppler Effect due to the proper motion gets virtually negligible. For such objects the distance is usually expressed as $z$ -value, which can be easily determined by the measured and heliocentrically corrected Red Shift [30].

$$z = \frac{\Delta \lambda}{\lambda_0}$$  \{17\}

In this extreme distance range $z$ replaces usually the absolute distance information, because this value is easy to determine from the spectrum and independent of any cosmological models. Due to the finite and constant speed of light, $z$ is also used as a measure of
the past. In contrast to the determination of \( z \), the "absolute" distance \( D \) is dependent on further parameters. Formula \( \{18\} \) allows a rough distance estimate with the so-called Hubble Parameter \( H \). This requires first, to express the cosmological Red Shift \( z \) in formula \( \{19\} \), as an apparent, heliocentric "recession velocity" \( v_f = c \cdot z \) (using the Doppler Principle).

\[
v_f = H \cdot D \quad \{18\} \quad z = \frac{v_f}{c} \quad \{19\} \quad D = \frac{c \cdot z}{H} \quad \{20\}
\]

\( H \approx \text{ca. } 73\pm8 \text{ km s}^{-1} \text{ Mpc}^{-1} \) \( D \) = Distance in Megaparsec Mpc

Thus the apparent "recession velocity" \( \cdot z \) and the Red Shift of the spectrum grow proportionally with the distance to the galaxy \( \{20\} \). Today it is known that the Hubble Parameter \( H \), seen over time, doesn’t remain constant. This term has therefore displaced the historically used "Hubble constant" \( H_0 \). The current value for \( H \) was determined in the so-called \( H_0 \text{ Key Project} \), using the Hubble Space Telescope. The linear formulas \( \{18\} \) \( \{20\} \) are only applicable up to about \( D \approx 400 \) Mpc or \( z < 0.1 \) [431]. Larger distances require the use of cosmological models. Instead of formula \( \{19\} \) a "relativistic" formula must then be applied which takes into account the effects of SRT [7]. Otherwise for \( z > 1 \), the simplified formula \( \{19\} \) would yield \( v_f \) values greater than the speed of light \( (c) \)!

\[
Z = \sqrt{\frac{c + v_f}{c - v_f} - 1} \quad \{21\}
\]

For radial velocities \( >1000 \) km/s, instead of the conventional Doppler formula \( \{15\} \), also the relativistic version should be used, which takes into account the effects of SRT [7].

\[
v_r = c \cdot \frac{(z + 1)^2 - 1}{(z + 1)^2 + 1} \quad \{22\}
\]

The range of the observed \( z \)-values reaches at present (2010) up to the galaxy Abell 1835 IR 1916 with \( z = 10 \), discovered in 2004 by a French/Swiss research team with the VLT at ESO Southern Observatory.

Example:

Calculation of the cosmological related share of the apparent "recession velocity" for the Whirlpool galaxy M51, based on the known distance \( D \) from 30M ly (Karkoschka):

\( D = 30M \text{ ly} = 9.2 \text{ Mpc} \) according to formula \( \{18\} \) follows \( v_f \approx +672 \text{ km/s} \)

The share of the \( z \)-value for M51, which is caused by the cosmological redshift, is calculated with formula \( \{19\} \) to just \( z_{\text{cosm}} = 0.0022 \), which is, considered in the cosmic scale, still very low.

15.6 Short Excursus on "Hubble time" \( t_H \)

In this case, it is worthwhile to take a small excursus, since the Hubble parameter allows in a very simple way to estimate the approximate age of the universe! With the simplified assumption of a constant expansion rate of the universe after the "Big Bang", by changing formula \( \{18\} \), it can be estimated, how long ago the entire matter was concentrated at "one point". This time span is also called Hubble time \( t_H \) and is equal to the reciprocal of the Hubble Parameter \( H \). This reciprocal value also corresponds to the Division of distance \( D \) by the expansion or "recession velocity" \( v_f \) and thus the desired period of time \( t_H \)!

\[
t_H = \frac{1}{H_0} = \frac{D}{v_f} \quad \{23\}
\]
To calculate the “Hubble time” we just need to put the units of the Hubble Parameter in to equation (23) and to convert \( s \) to \( \text{[years]} \) and \( [\text{Mpc}] \) to \( \text{[km]} \).

\[
1 \, y = 3.15 \cdot 10^7 \, s \quad 1 \, \text{Mpc} = 3.09 \cdot 10^{19} \, \text{km}
\]

\[
t_0 = \frac{1}{73 \, \text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}} = \frac{s \cdot \text{Mpc}}{73 \, \text{km}} = \frac{\text{years} \cdot 3.09 \cdot 10^{19} \, \text{km}}{3.15 \cdot 10^7 \cdot 73 \, \text{km}} = 1.34 \cdot 10^{10} \text{ or } 13.4 \, \text{bn years}
\]

15.7 Radial- and Cosmological Recess Velocities of the Messier Galaxies

The table on the following page shows the measured heliocentric radial velocities \( v_r \) of the 38 Messier galaxies, sorted by increasing distance. They are compared with the distance-dependent cosmological recess velocities \( v_r \) and the corresponding \( z \)-values according to (18) (19). Source for the \( v_r \) values is the NED, NASA Extragalactic Database [101]. Positive values = Red Shift, negative values = Blue shift.

These figures clearly indicate that in this “immediate neighbourhood”, the kinematic proper motion of the galaxies still dominates. Nevertheless, the trend is already evident here that the measured radial velocities \( v_r \) tend with increasing distance more and more to the cosmological "recession velocity". Anyhow in more than 60M ly distance two galaxies can be found (M90 and M98) with relatively strong negative values (blue coloured rows). This behaviour show 6 of 38, or about 16% of the Messier Galaxies. Most distant is M58 with about 68M ly.
<table>
<thead>
<tr>
<th>Messier Galaxy</th>
<th>Distance $D$ [M ly / Mpc]</th>
<th>Measured Radial-velocity $v_r$ [km/s]</th>
<th>Cosmologically caused Recession velocity $v_f$ [km/s]</th>
<th>Cosmologic share of $z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M31 Andromeda</td>
<td>2.5 / 0.77</td>
<td>$-300$</td>
<td>$+56$</td>
<td>$+0.00018$</td>
</tr>
<tr>
<td>M33 Triangulum</td>
<td>2.8 / 0.86</td>
<td>$-179$</td>
<td>$+63$</td>
<td>$+0.00021$</td>
</tr>
<tr>
<td>M96</td>
<td>11 / 3.37</td>
<td>$+897$</td>
<td>$+246$</td>
<td>$+0.00082$</td>
</tr>
<tr>
<td>M105</td>
<td>11 / 3.37</td>
<td>$+911$</td>
<td>$+246$</td>
<td>$+0.00082$</td>
</tr>
<tr>
<td>M108</td>
<td>11 / 3.37</td>
<td>$+699$</td>
<td>$+246$</td>
<td>$+0.00082$</td>
</tr>
<tr>
<td>M81</td>
<td>12 / 3.68</td>
<td>$-34$</td>
<td>$+269$</td>
<td>$+0.00090$</td>
</tr>
<tr>
<td>M82</td>
<td>12 / 3.68</td>
<td>$+203$</td>
<td>$+269$</td>
<td>$+0.00090$</td>
</tr>
<tr>
<td>M83</td>
<td>15 / 4.60</td>
<td>$+513$</td>
<td>$+336$</td>
<td>$+0.0011$</td>
</tr>
<tr>
<td>M109</td>
<td>15 / 4.60</td>
<td>$+1048$</td>
<td>$+336$</td>
<td>$+0.0011$</td>
</tr>
<tr>
<td>M94</td>
<td>16 / 4.91</td>
<td>$+308$</td>
<td>$+358$</td>
<td>$+0.0012$</td>
</tr>
<tr>
<td>M84</td>
<td>17 / 5.21</td>
<td>$+1060$</td>
<td>$+380$</td>
<td>$+0.0013$</td>
</tr>
<tr>
<td>M64</td>
<td>24 / 7.36</td>
<td>$+408$</td>
<td>$+537$</td>
<td>$+0.0018$</td>
</tr>
<tr>
<td>M106</td>
<td>24 / 7.36</td>
<td>$+448$</td>
<td>$+537$</td>
<td>$+0.0018$</td>
</tr>
<tr>
<td>M101</td>
<td>27 / 8.28</td>
<td>$+241$</td>
<td>$+604$</td>
<td>$+0.0020$</td>
</tr>
<tr>
<td>M51 Whirlpool</td>
<td>30 / 9.2</td>
<td>$+600$</td>
<td>$+672$</td>
<td>$+0.0022$</td>
</tr>
<tr>
<td>M65</td>
<td>30 / 9.2</td>
<td>$+807$</td>
<td>$+672$</td>
<td>$+0.0022$</td>
</tr>
<tr>
<td>M66</td>
<td>30 / 9.2</td>
<td>$+727$</td>
<td>$+672$</td>
<td>$+0.0022$</td>
</tr>
<tr>
<td>M104 Sombrero</td>
<td>30 / 9.2</td>
<td>$+1024$</td>
<td>$+672$</td>
<td>$+0.0022$</td>
</tr>
<tr>
<td>M95</td>
<td>33 / 10.1</td>
<td>$+778$</td>
<td>$+737$</td>
<td>$+0.0025$</td>
</tr>
<tr>
<td>M74</td>
<td>35 / 10.7</td>
<td>$+657$</td>
<td>$+781$</td>
<td>$+0.0026$</td>
</tr>
<tr>
<td>M63</td>
<td>37 / 11.3</td>
<td>$+504$</td>
<td>$+825$</td>
<td>$+0.0027$</td>
</tr>
<tr>
<td>M102</td>
<td>44 / 13.5</td>
<td>$+757$</td>
<td>$+986$</td>
<td>$+0.0033$</td>
</tr>
<tr>
<td>M88</td>
<td>47 / 14.4</td>
<td>$+2281$</td>
<td>$+1051$</td>
<td>$+0.0035$</td>
</tr>
<tr>
<td>M89</td>
<td>50 / 15.3</td>
<td>$+340$</td>
<td>$+1117$</td>
<td>$+0.0037$</td>
</tr>
<tr>
<td>M87</td>
<td>52 / 16.0</td>
<td>$+1307$</td>
<td>$+1168$</td>
<td>$+0.0039$</td>
</tr>
<tr>
<td>M49</td>
<td>53 / 16.3</td>
<td>$+997$</td>
<td>$+1190$</td>
<td>$+0.0040$</td>
</tr>
<tr>
<td>M60</td>
<td>55 / 16.8</td>
<td>$+1117$</td>
<td>$+1226$</td>
<td>$+0.0041$</td>
</tr>
<tr>
<td>M86</td>
<td>56 / 17.1</td>
<td>$-244$</td>
<td>$+1248$</td>
<td>$+0.0042$</td>
</tr>
<tr>
<td>M100</td>
<td>56 / 17.1</td>
<td>$+1571$</td>
<td>$+1248$</td>
<td>$+0.0042$</td>
</tr>
<tr>
<td>M59</td>
<td>60 / 18.4</td>
<td>$+410$</td>
<td>$+1343$</td>
<td>$+0.0045$</td>
</tr>
<tr>
<td>M61</td>
<td>60 / 18.4</td>
<td>$+1566$</td>
<td>$+1343$</td>
<td>$+0.0045$</td>
</tr>
<tr>
<td>M77</td>
<td>60 / 18.4</td>
<td>$+1137$</td>
<td>$+1343$</td>
<td>$+0.0045$</td>
</tr>
<tr>
<td>M85</td>
<td>60 / 18.4</td>
<td>$+729$</td>
<td>$+1343$</td>
<td>$+0.0045$</td>
</tr>
<tr>
<td>M90</td>
<td>60 / 18.4</td>
<td>$-235$</td>
<td>$+1343$</td>
<td>$+0.0045$</td>
</tr>
<tr>
<td>M98</td>
<td>60 / 18.4</td>
<td>$-142$</td>
<td>$+1343$</td>
<td>$+0.0045$</td>
</tr>
<tr>
<td>M99</td>
<td>60 / 18.4</td>
<td>$+2407$</td>
<td>$+1343$</td>
<td>$+0.0045$</td>
</tr>
<tr>
<td>M91</td>
<td>63 / 19.3</td>
<td>$+486$</td>
<td>$+1409$</td>
<td>$+0.0047$</td>
</tr>
<tr>
<td>M58</td>
<td>68 / 20.9</td>
<td>$+1519$</td>
<td>$+1526$</td>
<td>$+0.0051$</td>
</tr>
</tbody>
</table>
15.8 Recession Velocity of the Quasar 3C273

These very low z-values clearly show that Messier’s world of galaxies still belongs to our “backyard” of the universe. As a contrast, the following composite spectrum demonstrates the impressively red shifted H-emission lines of the apparently brightest quasar 3C273 in the constellation Virgo.

Source: University of California

The following example shall present the practical aspects of these theories and demonstrate that the above formulas are not just “gimmicks at an academic level”. Along with many other amateurs the American John Blackwell has managed to measure the Red Shift of 3C273 with a Celestron C8 (!) and a Rainbow Optics Transmission grating with some 200 lines/mm. He measured with Vspec the Hβ emission line of the quasar. For details see [480]. The resolution as well as the signal to noise ratio of the spectrum plays only a marginal role. For this purpose just the position of prominent H-Balmer lines must be made visible and measurable.

Rough calculation without taking relativistic effects into account

The distance to this object is in fact already beyond the limit, where even for amateur purposes, the calculation with the non-relativistic formula set and without the application of cosmological model parameters, is tolerable {15 20}. Anyway the brightness of this quasar is certainly a borderline case with respect to the current state of amateur spectroscopy. A slit spectrograph like DADOS on an 8-inch telescope would be overstrained, since the brightness of 3C273 fluctuates between ~11m and 13m. For such faint objects, the transmission grating is very advantageous because there is no need to place and guide the object on the slit of the spectrograph.

John Blackwell reported that the measured wavelength of the Hβ emission line is \( \lambda = 5640.34\text{Å} \). Compared to \( \lambda_0 = 4861.33\text{Å} \) it is therefore red-shifted by \( \Delta \lambda = +779\text{Å} \). Calculated with formula (17), this gives a z-value for the Red Shift of \( z = \Delta \lambda / \lambda_0 = 0.1603 \). According to NED, NASA Extragalactic Database, the accepted value for 3C273 is 0.1583, what is remarkably consistent with Blackwell’s measurement!
This enormous distance allows to equalise $Vr$ with the apparent "recession velocity" $v_f$, because the kinematic proper motion of this object is no more relevant. According to formula \eqref{eq:19}, $v_f = c \cdot z \approx 48'000 \ km/s$. The distance estimation depends from the Hubble Parameter. Formula \eqref{eq:18} and the assumption for $H \approx 73$ yields a distance of $D = v_f/H \approx 657\ Mpc \approx 2.14 \ bn\ ly$. The accepted value is $\approx 2.4bn\ ly$.

Keep in mind: The light which we analyse today from 3C273 was “on the road” about half as long like the age of our solar system! But compared with the $z \approx 10$ for Abell 1835, this is still relatively "close". This example also shows why the current and future space telescopes (eg Herschel, James Webb) are optimised for the infrared range.
16 The Measurement of the Rotation Velocity

16.1 Terms and Definitions

The Doppler shift $\Delta \nu$ due to the different radial velocities of the eastern and western limb of a rotating spherical celestial body, allows the determination of the rotational surface speed. Here we limit ourselves to the spectroscopic directly measurable portion of the rotation speed, the so-called $v \sin i$ value, which is projected into the visual line to Earth.

This term, read as a formula, allows calculating this velocity share with given effective equatorial velocity $v$ and the inclination angle $i$ between the rotation axis and the visual line to Earth. The Wikipedia graph shows $\nu$ here differently as $\nu_c$.

If the rotation axis is perpendicular to the sight line to Earth then $i = 90^\circ$ and $\sin i = 1$. Exclusively in this special case, we can exactly measure the equatorial velocity $V$.

If $i = 0^\circ$, we look directly on a pole of the celestial body and thus the projected rotational velocity becomes $\sin i = 0$.

16.2 The Rotation Velocity of the Large Planets

If the slit of the spectrograph is aligned with the equator of a rotating planet, the absorption lines of the reflected light appear slightly leaning. This is caused by the Doppler shift due to the radial velocity difference $\Delta \nu$ between the eastern and western limb of the celestial body. The rough alignment of the slit can be done by Jupiter with help of its moons and by Saturn with its ring.

The radial velocity difference $\Delta \nu$ is calculated from the obliquity of the spectral lines. For this purpose from a good quality spectrum two narrow strips, each on the lower and upper edge are processed and calibrated in wavelength. The difference between the upper and lower edge gives the Doppler Shift $\Delta \lambda$ and with formula (15) or (22) the velocity difference $\Delta \nu$ can be obtained (detailed procedure, see [30]).
16.3 The Rotation Velocity of the Sun
This method allows also the estimation of the Sun’s rotational surface speed, of course with an attached energy filter (!). This velocity however is so low (~2 km/s), that a high resolution spectrograph is required. In this case, the picture of the Sun on the slit plate would be too large, respectively the slit much too short to cover the entire solar equator. Required in such cases is the recording of two separate wavelength-calibrated spectra, each on the east- and west-limb of the Sun. This method was first practiced 1871 by Hermann Vogel.

16.4 The Rotation Velocity of Galaxies
This method can also be applied to determine the rotation velocity in the peripheral regions of galaxies. This works only with objects, which we see nearly “edge on” eg M31, M101 and M104 (Sombrero). For Face On galaxies like M51 (Whirlpool), we can just measure the radial velocity according to sect. 15.

16.5 Calculation of the $v \sin i$ Value with the Velocity Difference $\Delta v$
Here two cases must be distinguished:
1. Light-Reflective Objects of the Solar System:
   For light-reflecting objects of our solar system, eg planets or moons, the Doppler Effect seen from earth, acts twice. A virtual observer at the western limb of the planet sees the light of the sun (yellow arrow) as already red shifted by an amount, corresponding to the radial velocity of this point, relative to the Sun. This observer also notes that this light is reflected unchanged towards Earth with the same red shift (red arrow).
   An observer on Earth measures this reflected light, red-shifted by an additional amount, which is equal to the radial velocity of the western limb of the planet, relative to the Earth. This total amount is too high by the share, the virtual observer on the western limb of the planet has already found, in the incoming sunlight. When the outer planets are close to the opposition, both red-shifted amounts are virtually equal. A halving of the measured total amount yields therefore the velocity difference $\Delta v$ between the eastern and western limb of the planet. This velocity difference $\Delta v$ must now be halved again to obtain the desired, rotation velocity $v \sin i$. This corresponds then finally to $\frac{1}{4}$ of the originally measured red shift ($\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$).

   \[ v \sin i = \frac{\Delta v}{4} \quad \{24\} \]
   \[ \Delta v = \text{Calculated velocity difference} \]

2. Self-Luminous Celestial Bodies
   For self-luminous celestial objects, e.g. the Sun or galaxies, only a halving of the measured velocity difference is required:

   \[ v \sin i_{\text{Self}} = \text{Projected rotation velocity of self-luminous objects} \quad v \sin i = \frac{\Delta v}{2} \quad \{25\} \]
   \[ \Delta v = \text{Calculated velocity difference} \]
16.6 The Rotation Velocity of the Stars

Due to the huge distances, with few exceptions, even with large telescopes, stars can’t be seen as discs, but just as small Airy disks due to diffraction effects in optics. In this case the above presented method therefore fails and remains reserved for 2-dimensional appearing objects. Today, numerous methods exist for determining the rotational speed of stars, eg with photometrically detected brightness variations or by means of interferometry. The idea to gain the projected rotational velocity $v \sin i$ from the spectrum is almost as old as the spectroscopy itself.

William Abney has proposed already in 1877 to determine $v \sin i$ analysing the rotationally induced broadening of spectral lines. This phenomenon is also caused by the Doppler Effect, because the spectrum is composed of the light from the entire stellar surface, facing us. The broadening and flattening of the lines, is created by the rotationally caused different radial velocities of the individual surface points. This so-called "rotational broadening" is not the only effect that influences the FWHM of the spectral line (sect. 7.2 and 13.9). A successful approach was to isolate the Doppler Broadening-related share with various methods, eg by comparison with synthetically modelled spectra or slowly rotating standard stars. The graph on the right [52] shows this influence on the shape of a Mg II line at 4481.2 Å, for a star of the spectral class A.

The numerous measured rotational velocities of main sequence stars show a remarkable behaviour in terms of spectral classes. The graph shows a decrease in speed from early to late spectral types (Slettebak).

From spectral type G and later, $v \sin i$ amounts only to a few km/s (Sun ~2 km / s). The entire speed range extends from 0 to >400 km/s. It has also shown that stars after leaving the Main Sequence on their way to the Giant Branch in the HR diagram (sect. 14) as expected, greatly reducing the rotation speed.

Since the $v \sin i$ values get very low from the spectral class G on and later (so-called slow rotators), the demand on the resolution of the spectrograph is drastically increased. The focus of this process, particularly for amateurs, is therefore on the early spectral classes O - B, where the so-called fast rotators dominate. A typical example is Regulus, B7V, with $v \sin i = 317 \text{ km/s}$. The shape of this star is thereby strongly flattened. But there are also outliers. Sirius for example, as a representative of the early A-Class (A1V), is with 16 km/s a clear slow rotator [126].

An interesting case is Vega (A0V) with $v \sin i = 20 \text{ km/s}$, which for a long time has also been considered as an outlier. But various studies, e.g. Y. Takeda et al. [120], have shown that Vega is likely a fast rotator, where we look almost exactly on a pole ($i \approx 7^\circ$). This is supported by interferometrically detected, rotationally induced darkening effects (Peterson, Aufdenberg et al. 2006). The spread of the newly estimated effective $v$ – values for Vega is broad and ranges in these studies of approximately 160 - 270 km/s. This example clearly
shows the difficulties, to determine the inclination $i$ and the associated effective rotation velocity $v$, – this compared with the relatively simply determinable $v \sin i$ value! Today the FWHM - Method is complemented e.g. by a Fourier analysis of the line profile. The first minimum point represents here the value for $v \sin i$ with a resolution of some 2 km/s [125]. This method requires high-resolution spectra.

Various studies have shown, that the orientation of the stellar rotation axes is randomly distributed – in contrast to most of the planets in our solar system axes. Since the effective equatorial velocity can be determined only in exceptional cases, the research is here limited almost exclusively on statistical methods, based on extensive $v \sin i$ - samples. Most amateurs will probably limit themselves to reproduce well known literature values with high rotation velocities, typically for the earlier spectral classes.

**Empirical Formulas for $v \sin i$ depending on FWHM, $v \sin i = f(FWHM)$**

A considerable number of astrophysical publications in the SAO/NASA database (~1920 to the presence) deal with the calibration of the rotational speed, relative to FWHM. Some of the numerous "protagonists" are here A. Slettebak, O. Struve, G. Shajn, F. Royer and F. Fekel. Probably the most cited standard work in this respect is the so called „New Slettebak System” from 1975. Anyway recent studies have shown that the provided $v \sin i$ values are systematically too low. That's why I present here more recent approaches. The variable in most such formulas is $FWHM_{corr}$, given in [Å] {3}. But in some formulas $FWHM_{corr}$ is also expressed as a Doppler velocity [km/s]. In one case, also the Equivalent Width EW [Å] is involved (sect. 7).

**The Method according to F. Fekel**

This well established method [122, 123] is based on two different calibration curves, each one for the red and blue region of the spectrum. The two polynomial equations of the second degree calibrate the "raw value" for $v \sin i$ in [km/s], relating to the measured and adjusted $FWHM_{corr}$ in [Å]. Below are the two calibration polynomials (26a, 27a) according to Fekel, for the spectral ranges at 6430 Å and 4500 Å, followed by two further formulas (26b, 27b), which I have transformed from equations (26a, 27a) according to the explicitly requested $X - $ Values:

$$FWHM_{corr\, 6430} \approx 0.04082 + 0.02509X + 0.00014X^2$$  \quad {26a}$$

$$FWHM_{corr\, 4500} \approx 0.08016 + 0.01284X + 0.00011X^2$$  \quad {27a}$$

$$X_{6430} \approx \sqrt{7143 \cdot (FWHM_{corr\, 6430} + 1.08)} - 89.6$$  \quad {26b}$$

$$X_{4500} \approx \sqrt{9091 \cdot (FWHM_{corr\, 4500} + 0.29)} - 58.1$$  \quad {27b}$$

The procedure for the determination of $v \sin i$ is described in detail and supported by an example in [30]. Here follows a short overview:

First, several FWHM values [Å] are measured and averaged by weakly to moderately intense and Gaussian fitted spectral lines (no H-Balmer lines). Further these values are cleaned from the "instrumental broadening" ($FWHM_{corr}$). Then the X values are calculated by inserting the $FWHM_{corr}$ values in the above formulas (26b) or (27b), according to the wavelength ranges at 4500 Å or 6430 Å. These X-values must then be cleaned from the line broadening due to the average macro turbulence velocity $v_m$ in the stellar atmosphere, using the following formula. This will finally yield the desired $v \sin i$ value [km/s]

$$v \sin i = \sqrt{X^2 - v_m^2}$$  \quad {28}
The variable $v_m$ depends on the spectral class. For the B- and A-class Fekel assumed $v_m = 0$. For the early F-classes $v_m = 5 \text{ km/s}$, Sun like Dwarfs $v_m = 3 \text{ km/s}$, K-Dwarfs $v_m = 2 \text{ km/s}$, early G-Giants $v_m = 5 \text{ km/s}$, late G- and K- Giants $v_m = 3 \text{ km/s}$ and F – K Sub Giants $v_m = 5 – 3 \text{ km/s}$.

Below, the commonly used spectral lines for determining the FWHM values are listed. The lines, proposed by Fekel are in bold italics, such used by other authors only (eg Slettebak) are written in italics. The supplement “(B)” means that the profile shape is blended with a neighbouring line. “(S)” means a line deformation due to an electric field (Stark Effect):

- **Late F-, G- and K- spectral classes**: Analyse of lines preferably in the range at 6430Å:
  - eg $\text{Fe II} 6432, \text{Ca I} 6455, \text{Fe II} 6456, \text{Fe I} 6469, \text{Ca I} 6471$.

- **Middle A-Classes and later**: Analyse of moderately intense $\text{Fe I}, \text{Fe II}$ and $\text{Ca I}$ lines in the range at 6430Å. For A3 – G0-Class $\text{Fe I} 4071.8$ (B) und $4072.5$ (B).

- **O–, B– and early A–classes**: Analyse of lines in the range at 4500 Å:
  - several $\text{Fe II}$ and $\text{Ti II}$ lines, as well as $\text{He I} 4471$ and $\text{Mg II} 4481.2$.

- **O–, early B– and Be– classes**: $\text{He I} 4026$ (S), $\text{Si IV} 4089$, $\text{He I} 4388$, $\text{He I} 4470/71$ (S,B), $\text{He II} 4200$ (S), $\text{He II} 4542$ (S), $\text{He II} 4686$, $\text{Al III}$, $\text{N II}$.

As an alternative to F. Fekel, A. Moskovitz [121] in connection with K-Giants, analysed exclusively the relatively isolated $\text{Fe I}$ line at 5434.5 with formula {26a}, respectively {27a}.

### 16.7 The Rotation Velocity of the Circumstellar Disks around Be Stars

Be stars form a large subgroup of spectral type B. *Gamma Cassiopeiae* became the first Be star discovered in 1868 by Father Secchi (sect. 13.3), who wondered about the "bright lines" in this spectrum. The lower case letter e (Be) already states that here appear emission lines.

In contrast to the stars, which show P Cygni profiles due to their expanding matter (sect. 17), it is here in most cases a just temporarily formed, rotating circumstellar disk of gas in the equatorial plane of the Be star. Their formation mechanisms are still not fully understood. This phenomenon is accompanied by hydrogen emission, and strong infrared- as well as X-ray radiation. Outside of such episodic phases, the star leads a seemingly normal life in the B-class. In research and monitoring programs of these objects, many amateurs are involved with photometric and spectroscopic monitoring of the emission lines (particularly Hα). That's why I've gathered some information about this exciting class, since here also applications for the EW and FWHM values can be presented. Here follow some parameters of these objects, based on lectures given by Miroshnichenko [140], [141], as well as publications by K. Robinson [5] and J. Kaler [3]:

- 25% of the 240 brightest Be stars have been identified as binary systems.
- Most Be stars are still on the Main Sequence of the HRD, spectral classes O1–A1 [141]. Other sources mention the range O7–F5 (F5 for shell stars).
- **Be stars** consistently show high rotational velocities, up to >400 km/s, in some cases even close to the so-called "Break Up" limit. The main cause for the spread of these values should be the different inclination angles of the stellar rotation axes.
- The cause for the formation of the circumstellar disk and the associated mass loss is still not fully understood. In addition to the strikingly high rotational speed [3], etc. it is also attributed to non-radial pulsations of the star or the passage of a close binary star component in the periastron of the orbit.

- Such discs can arise within a short time but also disappear. This phenomenon may pass through overall three stages: Common B star, Be–star and Be–shell star [33]. Classic example for that behaviour is Plejone (Pleiades M45), which has passed all three phases within a few decades [147]. Due to its viscosity, the disk material moves outwards during the rotation [141].

- With increasing distance from the star, the thickness of the disk is growing and the density is fading.

- If the mass loss of the star exceeds that of the disc, the material is collected close to the star. In the opposite case it can form a ring.

- The X-ray and infrared radiation is strongly increased.

If a B star mutates in a relatively short time to a Be star (e.g. scorpii δ), the Hα absorption from the stellar photosphere is changing to an emission line, now generated in the circumstellar disk. At the same time it becomes the most intense spectral feature (extensive example see [30] sect. 22). It represents now the kinematic state of the ionised gas disk. It is, similar to the absorption lines of ordinary stars, broadened by Doppler Effects, but here due to the rotating disk of gas and additional, non-kinematic effects. Therefore the FWHM value of the emission line is now a measure for the typical rotation velocity of the disc material. For δ scorpii in [146] it is impressively demonstrated, that since about 2000 the FWHM and EW values of the Hα emission line are subject to strong long-period fluctuations. Between the first outbursts in 2000 to 2011, the Doppler velocity of the FWHM value fluctuated between about 100–350 km/s and the EW value from -5 to -25 Å, indicating highly dynamic processes in the disk formation process. Furthermore the chronological profile of FWHM and EW values is strikingly phase-shifted.

**Formula for the Rotation Velocity of the Disk Material:**

Several formulas have been published, which allow to estimate the rotation speed of the disc material from Be stars. In most cases $v \sin i$ is calculated with $FWHM_{corr}$ values at the Hα line.

Following a formula by Dachs et al which Soria used in [145]: It expresses explicitly the $v \sin i$ value, based on the $FWHM_{corr}$ [km/s] at the Hα emission line, combined with the (negative) equivalent width EW [Å].

$$v \sin i \ (±30 \text{ km/s}) \approx \frac{FWHM_{corr \ Hα}}{2} \cdot \left[ \frac{EW_{Hα}}{-3\text{Å}} \right]^\frac{1}{4} - 60 \text{ km/s} \quad \{29\}$$

**Example:** The Hα Line of Soria’s Be–star yields $FWHM_{corr \ Hα} = 6.1$ Å, corresponding to a Doppler Velocity of 278 km/s and $EW = -16$ Å. This results to $v \sin i = 151$ km/s.

In [30] sect. 22.3 this formula is applied to a DADOS spectrum of δ scorpii. The accuracy of this method is limited to ±30 km/s. Therefore "estimate", is here probably the better expression than "calculate". Hanuschik [127] shows a simple linear formula, which expresses $v \sin i$ just with the $FWHM_{corr \ Hα}$ [km/s] of the Hα emission line. It corresponds to the median fit of a strongly scattering sample with 115 Be stars, excluding those for which an underestimation of the $v \sin i$ value was assumed; [127], (1b).

$$v \sin i \approx \frac{FWHM_{corr \ Hα} - 50 \text{ km/s}}{1.4} \quad \{30\}$$
With Soria’s $FWHM_{Korr, H_\alpha} = 6.1 \, \text{Å}$ (see above) results a different $v \sin i = 163 \, \text{km/s}$

The following formula is applicable for the $FWHM_{Korr}$ $v$-values at the emission lines $H_\beta$ (4861.3) and $FE\, II$ (5317, 5169, 6384, 4584 Å).

$$v \sin i \approx \frac{FWHM_{Korr, H_\beta, \, \text{ell}} - 30 \, \text{km/s}}{1.2}$$  \quad (31)

The Distribution of the Rotation Velocity on the Disk

Assuming that the disk rotation obeys the kinematic laws of Kepler, the highest rotational velocity $v$ occurs on its inner edge – In many cases, probably identical with the star equator. It decreases towards the outside (formula according to Robinson [5]).

$$v = \sqrt{\frac{G \cdot M}{R}}$$  \quad (33)

$G = \text{Gravitation constant} \, [6.674 \cdot 10^{-11} \cdot m^3 \cdot kg^{-1} \cdot s^{-2}]$

$M = \text{Mass of the star} \, [kg]$

$R = \text{Distance from the considered disk part to the star center} \, [m]$

The application of this formula is limited to high $v \sin i$ values ($v \sin i \approx v$) or known inclination angle $i$.

The Analysis of Double Peak Profiles

The emission lines of Be stars often show a double peak. The gap between the two peaks is explained with the Doppler-, self-absorption- and perspective effects. Looking at the edge of the disk in the range of the symmetry axis the gas masses apparently move perpendicularly to our line of sight, i.e. the radial velocity there is $v_r \approx 0$. Indicated are important dimensions, which are used in the literature.

$V = \text{Intensity} \, (V/I_c) \, \text{violet shifted Peak}$

$R = \text{Intensität} \, (R/I_c) \, \text{red shifted Peak}$

$I_c = \text{Normalised continuum level} = 1$

$V/R = \text{Peak Intensity ratio}$

$\Delta v_{\text{peak}} = \text{Distance between the Peaks} \, [\text{km/s}]$

Distance between the Peaks $\Delta v_{\text{peak}}$

The chart on the right shows according to K. Robinson [5] the modelled emission lines for different inclination angles $i$ (sect. 16.1). It seems obvious that the distance $\Delta v_{\text{peak}}$ increases with growing inclination $i$. At the same time also the $v \sin i$ values are increasing, if for all inclination angles a similar effective equatorial velocity $v$ and a fixed disc radius $R_s$ are assumed. $\Delta v_{\text{peak}}$ is expressed as a velocity value according to the Doppler principle: according to [15]:

$$\Delta v_{\text{peak}} \, [\text{km/s}] = \Delta v_{\text{peak}} \, [\text{Å}] \cdot c/\lambda_0$$
Also according to Hanuschik [143] a rough correlation exists between the $\Delta v_{\text{peak}}$ [km / s] at the Hα line and $v \sin i$.

$$\Delta v_{\text{peak}, \text{Hα}} \approx (0.4 ... 0.5) \cdot v \sin i \quad \{34\}$$

According to Miroshnichenko [141] and Hanuschik [143] a decreasing disk radius $R$ is also associated with an increasing $\Delta v_{\text{peak}, \text{Hα}}$. This statement is consistent with formula {33} and {34}.

The outer disc radius $R_s$

The formula by Huang [143] allows the estimation of the outer disk radius $R_s$, expressed as [stellar radii r], based on $\Delta v_{\text{peak}}$ and the velocity $v$ at the inner edge of the disc, probably touching the star’s equator in most of the cases ($r = 1$).

$$R_s \approx \left( \frac{2 \cdot v}{\Delta v_{\text{peak}}} \right)^2 \quad \{35\}$$

The application of this formula is limited to high $v \sin i$ values ($v \sin i \approx v$) or known inclination angle $i$.

Peak Intensity Ratio $V/R$ (Violet/Red)

The V / R ratio is one of the main criteria for the description of the double peak emission of Be stars. S. Stefl et al. [144] have supervised this ratio for about 10 years, at a sample of Be stars. They have noted long-periodic variations of 5-10 years. One of the discussed hypotheses are oscillations in the inner part of the disk.

At δ scorpii the V / R ratio of the He I line (6678.15 Å) shows a strong variation since the outbreak of 2000 [146]. See also example in [30] sect. 22.3.

According to Kaler [3] the V/R ratio also reflects the mass distribution within the disc and may show a rather irregular course.

In Be-Binary systems a pattern of variation seems to occur, which is linked to the orbital period of the system.

According to Hanuschik [143] asymmetries of the emission lines, eg $V/R \neq 1$, are related to radial movements and darkening effects.

If no double peak is present in the emission line, according to [128] the asymmetry in the steepness of the two edges can be analysed. $V>R$ means the violet edge is steeper and vice versa by $R>V$ the red one.
17. The Measurement of the Expansion Velocity

Different types of stars repel at certain stages of evolution more or less strongly matter. The speed range reaches from relatively slow 20–30 km/s, typical for planetary nebulae, up to several 1000 km/s for Novae and Supernovae (SNR). This process manifests itself in different spectral symptoms, dependent mainly on the density of repelled material.

17.1 P Cygni Profiles

The P Cygni profiles have been already introduced in sect. 5.5, as examples for mixed absorption- and emission line spectra. They are a common spectral phenomenon which occurs in all spectral classes, and are a reliable sign for expanding star material. The evaluation of this effect with the Doppler formula is demonstrated here at the expansion velocity of the stellar envelope of P Cygni.

\[ \nu_r = \frac{\Delta \lambda}{\lambda_0}, c \quad \{15\} \]

The offset \( \Delta \lambda [\text{Å}] \) is measured between the emission and the blueshifted absorption part of P Cygni profiles. In the example for the H\( \alpha \) line of P Cygni itself, the measured difference yields:

\[ \Delta \lambda = \lambda_{Blue} - \lambda_{Red} = 6557.7 - 6562.2 = -4.5 \text{ Å} \]

(single measurement of 07.16.2009, 2200 UTC, DADOS and 900L/mm grating)

with \( \lambda_0 = 6562.82 \text{ Å}, c = 300'000 \text{ km/s}, \{15\} \) yields:

\[ \nu_r = V_{Envelope} \approx -206 \text{ km/s} \]

The accepted values lie in the range of –185 bis – 205 km/s ±10km/s. The heliocentric correction is not necessary here, since the Doppler shift is measured not absolutely but relatively, as the \( \Delta \lambda \) difference, visible in the spectrum. P Cygni forms not a separate class of stars. Such profiles are a common spectral phenomenon, which can be found in all spectral classes and are a reliable indication for expanding stellar matter.

17.2 Inverse P Cygni Profiles

In contrast to the normal P Cygni profiles, the inverse ones are always a reliable sign for a contraction. The absorption kink of this feature is here shifted to the red side of the emission line. Textbook example is the Protostar T Tauri which is formed by accretion from a circumstellar gas and dust disk. The forbidden [O I] und [S II] lines show here clearly inverse P Cygni profiles, indicating large-scale contraction movements within the accretion disk, headed towards the protostar. The Doppler analysis showed here contraction velocities of some 600 km/s. The excerpt of the T Tauri spectrum is from [33], Table 18.
17.3 Broadening of the Emission Lines

P Cygni profiles are not only characteristic for stars with strong expansion movements, but also for Novae and Supernovae. Anyway such extreme events show more often just a strong broadening of the emission lines. This applies also to Wolf Rayet stars, even though with significantly lower FWHM values (see [33]). According to [160] the expansion velocity can be estimated, by putting $FWHM_{Emission\ H\alpha}$ in to the conventional Doppler formula, instead of $\Delta \lambda$:

$$v_r \approx \frac{FWHM_{Emission\ H\alpha}}{\lambda_0} \cdot c \quad \{36\}$$

The scheme [160] shows for the Nova V475 Scuti (2003) four developmental phases within 38 days. Here, nearly the tenfold expansion velocity of the P Cygni occurred, already an order of magnitude at which the relativistic Doppler formula should be applied [22].

17.4 Splitting of the Emission Lines

In sect. 16.7 the determination of the rotation velocity by evaluating the double peak profiles in Be stars, was already presented. Split emission lines can also be observed in spectra of relatively old, strongly expanded and thus optically transparent stellar envelops. Textbook example is here SNR M1.

The chart at right explains the split up of the emission lines due to the Doppler Effect. The parts of the shell which move towards earth cause a blue shift of the lines and the retreating ones are red shifted. Thereby, they are deformed to a so-called velocity ellipse. This effect is seen here at the noisy [O III] lines of the M1 spectrum [33].

The chart at right shows the splitting of the H\alpha line in the central area of the Crab Nebula M1 ([33] Table 85). Due to the transparency of the SNR, with $\Delta \lambda$ the total expansion velocity, related to the diameter of the SNR is calculated (here about 1800 km/s). The radial velocity is obtained finally by halving this value. It yields therefore just below 1000km/s
18 Spectroscopic Binary Stars

18.1 Terms and Definitions

> 50% of the stars in our galaxy are gravitationally connected components of double or multiple systems. They concentrate primarily within the spectral classes A, F and G [170]. For the astrophysics these objects are also of special interest because they allow a determination of stellar masses independently of the spectral class. Soon after the invention of the telescope, visual double stars were also observed by amateur astronomers. Today the spectroscopy has opened for us also the field of spectroscopic binary stars.

An in-depth study of binary star orbits is demanding and requires eg celestial mechanics skills. Here should only be indicated, what can be achieved by spectroscopic means. Scientifically relevant results are usually only possible associated with long term astrometric and photometric measurements. Spectroscopic binary stars are orbiting in such close distances around a common gravity center, that they can’t be resolved even with the largest telescopes in the world. They betray their binary nature just by the periodic change in spectral characteristics. For such close orbits Kepler’s laws require short orbital periods and high-track velocities, significantly facilitating the spectroscopic observation of these objects.

In contrast to the complex behaviour of multiple systems, the motion of binary stars follows the three simple Kepler’s laws. Its components rotate at variable velocities \( v_M \) in elliptical orbits around a common Barycenter \( B \) (center of gravity). The following sketch shows a fictional binary star system with stars of the unequal sizes \( M_1 \) and \( M_2 \). For simplicity their elliptical orbits are running here exactly in the plane of the drawing as well as the sight line to Earth which in addition runs parallel to the minor semi-axes. For this perspective special case the orbital velocity \( v_M \) at the Apsastron (farthest orbital point) and Periastron (closest orbital point) corresponds also to the observed radial velocity \( v_r \). The recorded maximum values (amplitudes) are referred in the literature with \( K \). The following layout corresponds to an orbit- inclination of \( i = 90^\circ \) (sect. 18.3).
- Both orbital ellipse:
  - must be in the same plane
  - have different sizes depending on the mass
  - must be similar to each other, i.e. have the same eccentricity \( e = \sqrt{a^2 - b^2}/a \)
- The more massive star \((M_1)\) always runs on the smaller elliptical orbit and with the lower velocity \(v_M\) around the barycenter.
- The barycenter lies always in one of two focal points
  - \(M_1\) und \(M_2\) always run synchronously:
    - During the entire orbit the connecting line between \(M_1\) and \(M_2\) runs permanently through the barycenter
    - \(M_1\) and \(M_2\) always reach the Apastron as well as the Periastron at the same time.

### 18.2 Effects of the Binary Orbit on the Spectrum

Due to the radial velocities the Doppler shift causes striking effects in the spectrum. The above assumed perspective special case for the orbit orientation would maximize these phenomena for a terrestrial observer (see below phase \(D\)). Generally, two different cases can be distinguished \[180\].

1. **Double stars with two components in the spectrum – \(SB2\)-systems**

If the apparent brightness difference between the two components lies approximately at \(0 - 1^m\), we can record the composite spectrum of both stars. The following phase diagrams are based on the above assumptions and show these effects within one complete orbit:

Here the orbital velocities \(v_M\) are directed perpendicular to the line of sight and thus the radial velocity with respect to the Earth becomes \(v_r = 0\). The spectrum remains unchanged, i.e. \(\Delta \lambda = 0\).

In the Apastron the orbital velocities \(v_M\) are minimal. But they run now parallel to the line of sight, and correspond here to the radial velocities, so \(v_r = v_M\). The spectral line appears splitted:
\[
\Delta \lambda_A = (v_{rM1A} + v_{rM2A}) \cdot \lambda/c \quad \{37\}
\]

Here the orbital velocities \(v_M\) are directed perpendicular to the line of sight and thus the radial velocity with respect to the Earth becomes \(v_r = 0\). The spectrum remains unchanged, i.e. \(\Delta \lambda = 0\).

In the Periastron the orbital velocities \(v_M\) are maximal. They run now parallel to the line of sight, and correspond here to the radial velocities, so \(v_r = v_M = K\). The spectral line appears here more splitted compared to phase \(B\).
\[
\Delta \lambda_P = (v_{rM1P} + v_{rM2P}) \cdot \lambda/c \quad \{38\}
\]
The above introduced formulas for the Doppler shift $\Delta \lambda$ allow, after a simple change, to calculate the sum of the radial velocities $v_{rM1} + v_{rM2}$ with the line splitting $\Delta \lambda$. For the general case:

$$v_{rM1} + v_{rM2} = c \cdot \Delta \lambda / \lambda_0 \quad \{39\}$$

### The Calculation of the Individual Radial Velocities $v_{rM1}$ and $v_{rM2}$

If the mass difference is large enough, the splitting of the spectral line occurs *asymmetrically* with respect to the neutral wavelength $\lambda_{r0}$. With these uneven distances $\Delta \lambda_1$ and $\Delta \lambda_2$ and by analogy to (39) the individual radial velocities $v_{rM}$ can be calculated separately [172].

As a result of the heliocentric radial movement of the *entire star system* $v_{r\text{Syst}}$ [100], the wavelength of the "neutral", unsplit spectral line is shifted by the Doppler Effect from the stationary laboratory wavelength $\lambda_0$ to $\lambda_{r0}$ [170].

$$\lambda_{r0} = \lambda_0 + \frac{v_{r\text{Syst}}}{c} \cdot \lambda_0 \quad \{40\}$$

This is now the *adjusted reference point* for the measurement of the two distances $\Delta \lambda_{1,2}$. But first the $\lambda_{1,2}$ values must be heliocentrically corrected to $\lambda_{1,2\text{Hel}}$ according to [30], sect. 18, step 7. Then it follows:

$$\Delta \lambda_{1,2} = \lambda_{1,2\text{Hel}} - \lambda_{r0} \quad \{41\}$$

If no asymmetry occurs of the splitted line with respect to $\lambda_{r0}$, $M_1$ and $M_2$ are approximately equal and the sum of the two radial velocities $v_{rM1} + v_{rM2}$ just needs to be halved.

2. **Double stars with only one component in the spectrum – SB1–systems**

In most cases the apparent brightness difference between the two components is significantly $> 1^m$. Here with amateur equipment, only the spectrum of the brighter star can be recorded. Extreme cases are entirely invisible black holes as double star components, or extrasolar planets, which shift the spectrum of the orbited star by just a few dozen meters/sec! In such cases, a splitting of the line isn’t recognisable, but only the shift of $\lambda_1$ to the right or left in respect of the neutral position $\lambda_{r0}$.

The following example, recorded with DADOS 900L/mm shows this effect, using the spectroscopic A components within the quintuple system $\beta$ Scorpii. Impressively visible is here the Hα-shift of the brighter component, within three days. With this Vspec plot just this effect shall be demonstrated. A serious investigation of the orbital parameters would require the recording of multiple orbits at least at daily intervals! The lineshifts are plotted here with respect to $\lambda_{r0}$. The X-axis is scaled here in Doppler velocity, according to [30], sect. 19, and allows a rough reading of the radial velocity. The values $\lambda_{1,2}$ have been determined by Gaussian fit on the heliocentrically corrected profiles. Detailed procedure see [30] sect. 24.2.
Here are some orbital parameters of \( \beta \) Scorpii according to an earlier study by Peterson et al. [177]. These values are based on the measured, maximum radial velocities \( K_1 \) und \( K_2 \), obtained from the spectra of both components. The terms "\( \sin \ i \)" in the data list, and "\( \sin^3 \ i \)" for the masses, demonstrate that the inclination \( i \) is unknown and therefore the values are uncertain by this factor (for details see sect. 18.3 and 18.4).

- Spectral class of the brighter component: B0.5V, of the weaker component: unknown
- Orbital period: \( T = 6.83 \) Tage
- Stellar masses: \( M_1 \sin^3 i = 12.5 \, M_\odot, \quad M_2 \sin^3 i = 7.7 \, M_\odot \) [\( \odot \) = Solar Mass]
- Mass Ratio: \( M_1 / M_2 = 1.63 \)
- Max. recorded radial velocities: \( K_1 = 121 \, \text{km/s}, \quad K_2 = 198 \, \text{km/s} \)
- Major semi axes of the orbital ellipse: \( a_{M1} \sin i = 1.09 \cdot 10^7 \text{ km} \quad a_{M2} \sin i = 1.78 \cdot 10^7 \text{ km} \)

These figures show that the mass and brightness difference is substantial. With the large telescopes, involved in this study [177], the spectrum of the weaker component could still be recognised, but analysed just with substantial difficulties.

### 18.3 The Perspectivic Influence from the Spatial Orbit-Orientation

The orientation of the binary star orbital planes, with respect to our line of sight, shows a random distribution. The angle between the axis, perpendicularly standing on the orbital plane (normal vector), and our line of sight is called inclination \( i \) [175]. Thus the definition for the inclination of stellar and binary system rotation axes (sect. 16) is the same. Analogical, \( v_r \sin i \) is here the spectroscopically direct measurable share of the radial velocity \( v_r \), which is projected into the sight line to Earth. For \( i = 90^\circ \) we see the orbital ellipse exactly "edge on" i.e. \( \sin i = 1 \).

- This elliptical orbit, with a given and fixed inclination \( i \), may be rotated freely around the axis of the sight line, without any consequences for their apparent form.
- Thus for circular binary orbits the inclination \( i \) fixes the only degree of freedom, which affects the apparent shape of the orbit.
- For elliptical binary star orbits, the situation is more complex. In contrast to the circle, the orientation of the ellipse axes in a given orbital plane forms an additional degree of freedom, determining the apparent ellipse form.

If \( i \) remains unknown, the results can statistically only be evaluated, similar to the \( v \sin i \) values of the stellar rotation (sect. 16.6).

**Caveat:**

There are also reputable sources that define \( i \) as the angle between the line of sight and the orbital plane, similar to the inclination angle convention between planetary orbits and the ecliptic. Consequence: There must always be clarified which definition is used. The two conventions can easily convert to each other as complementary angle \( 90^\circ - i \).
18.4 The estimation of some orbital parameters

Based on purely spectroscopic observation some of the orbital parameters of the binary system can be estimated. First, the measured radial velocities \( v_{r,M1} \) and \( v_{r,M2} \) are plotted as a function of the time \( t \). The graph shows an orbital period of Mizar (\( \zeta \) Ursae Majoris, A2V), one of the showpieces for spectroscopic binaries with double lines (SB2 systems). Another similar example is \( \beta \) Aurigae with an orbital period of about 4 days. The more these curves show a sinusoidal shape, the lower is the eccentricity of the orbit ellipse [179].

The Orbital Period \( T \)

The orbital period \( T \) can directly be determined from the course of the velocity curves. As the only variable it remains largely unaffected by perspective effects and is thus relatively accurately determinable.

Simplifying to circular orbits

Since we are mostly confronted with randomly oriented, elliptical orbits, a reasonably accurate determination of orbital parameters is very complex. For this purpose, in addition to the spectroscopic, complementary astrometric measurement data would be needed. Only for eclipsing binaries, such as Algol, already a priori, a probable inclination of \( i > 80^\circ \) can be assumed. For the rough estimation of other parameters various sources suggest the simplification of the elliptical to circular orbits. Thus, the radii \( r_M \) and thus the orbital velocities \( v_M \) become constant. The mostly unknown inclination is expressed in the formulas with the term \( \sin i \).

The Orbital Velocity \( v_M \)

To determine the orbital velocity \( v_M \) we need from the velocity diagram, the maximum \( v_{r,M} \) values for both components. They correspond by definition, to the maximum amplitudes \( K_1 \) and \( K_2 \). For the circular orbit velocity \( v_M \) follows:

\[
K = v_M \cdot \sin i \quad \text{oder} \quad v_M = \frac{K}{\sin i} \quad \{42\}
\]

Determination of the orbital radii \( r_M \)

With the orbital period \( T \) and the velocity \( v_M = \frac{K}{\sin i} \) for a circular orbit the corresponding radius \( r_M \) can be calculated. From geometric reasons follows generally:

\[
T = \frac{2 \cdot r_M \cdot \pi}{K} \cdot \sin i \quad \{43\}
\]
\[ r_M = \frac{K \cdot T}{2 \cdot \pi \cdot \sin i} \]  \{44\}

If both lines can be analysed, then with the obtained \( K_1 \) und \( K_2 \) the corresponding radii \( r_{M1} \) and \( r_{M2} \) can be calculated separately.

**Calculation of the Stellar Masses \( M_1 + M_2 \) in SB2 Systems**

If both lines can be analysed, then with the following formula the total mass of the system \( M_1 + M_2 \), can be determined. This is obtained if \( \{44\} \), changed as a sum of radii \( r_{M1} + r_{M2} \), is used in the formula of the third Kepler's law, instead of the semi-major axis \( a = r_{M1} + r_{M2} \).

\[
M_1 + M_2 = \frac{T \cdot (K_1 + K_2)^3}{2 \cdot \pi \cdot G \cdot \sin^3 i} \]  \{45\}

\( G = \text{Gravitational constant} \ [6.674 \cdot 10^{-11} \cdot m^3 \cdot kg^{-1} \cdot s^{-2}] \).

The partial masses can then be calculated by their total mass \( M_1 + M_2 \) with the partial radii \( r_{M1} \) and \( r_{M2} \).

\[
M_1 \cdot r_{M1} = M_2 \cdot r_{M2} \]  \{46\}

\[
M_1 = \frac{r_{M2}}{r_{M1} + r_{M2}} (M_1 + M_2) \quad M_2 = \frac{r_{M1}}{r_{M1} + r_{M2}} (M_1 + M_2) \]  \{47\}

For binary star systems \( M \) is often expressed in solar masses \( M_\odot \) and the distance in AU.

To convert: \( 1 \text{ Solar mass } M_\odot \approx 1.98 \cdot 10^{30} kg \quad 1 \text{ AU} \approx 149.6 \cdot 10^9 m \)

**Calculation of the Stellar Masses \( M_1 + M_2 \) in SB1 Systems**

The analysis of only one line has logically consequences on the information content and accuracy of the system parameters to be determined. For SB1–systems in the spectrum, only the radial velocity curve of the brighter component \( M_1 \) can be analysed, and therefore just the amplitude \( K_1 \) can be determined. So with formula \( \{44\} \), only the corresponding orbital radius \( r_{M1} \) can be calculated. Unfortunately, neither their total mass nor their partial masses \( M_1, M_2 \) are directly determinable. Just the so called mass function \( f(M_1, M_2) \) can be determined [51 sect. VII], [179]. This expression is not really demonstrative and means the cube of the invisible partial mass \( (M_2^3) \) in relation to the square of the total mass \( (M_1 + M_2)^2 \). For a calculated example, see [171].

\[
f(M_1, M_2) = \frac{M_2^3 \cdot \sin^3 i}{(M_1 + M_2)^2} = \frac{T \cdot K_1^3}{2 \cdot \pi \cdot G} \]  \{48\}

It can be roughly determined eg by estimation of the visible mass \( M_1 \) by the spectral type. For a rough guide refer to the table in sect. 14.5. Such results still remain loaded with the "uncertainty" of \( \sin^3 i \).
19 Balmer-Decrement

19.1 Introduction

In spectra where the \( H \)-Balmer series occurs in emission, the line intensity \( I \) fades with decreasing wavelength \( \lambda \). This phenomenon is called the Balmer Decrement \( D \). The intensity loss is reproducible by the laws of physics and is therefore a highly important indicator for astrophysics. The hydrogen emission lines are formed by the electron transitions, which end, as well known, in "downward direction" on the second-lowest energy level \( n = 2 \). The probability, from which of the higher levels an electron comes, is determined for the individual lines by quantum mechanical laws. From this it follows that the intensity \( I \) is highest at the \( H\alpha \) line and gets continuously weaker at the shorter wavelength lines, \( H\beta, H\gamma, H\delta, H\epsilon \) etc. The extent of this decrease (decrement) is additionally, but only moderately, dependent on the density and temperature of the electrons \( N_e \) and \( T_e \).

19.2 Qualitative Analysis

For most amateurs probably qualitative applications of this effect are in the focus. On a low-resolution spectrum of P Cygni (DADOS 200L), the intensity-decrease of the dominant hydrogen emission lines is demonstrated.

Contrary to this trend Mira (\( o \) Ceti) shows only \( H\delta \), and significantly, \( H\gamma \) in emission. These impressive lines indicate what enormous intensities the \( H\alpha \) and \( H\beta \)-Balmer emissions should really show, according to the Balmer decrement. But these lines are covered in the long-wave region by titanium oxide absorption bands, which are apparently generated in higher layers of the star's atmosphere, as the \( H \) emission lines, details see [33].
19.3 Quantitative Analysis

Quantitatively, the Balmer decrement is mainly used for the spectroscopic determination of the interstellar extinction, the so-called "interstellar reddening". The "reddening" of the light occurs, because the blue part is absorbed or scattered stronger than the red. The steeper the decrement $D$ runs, compared to theoretically calculated values $D_{th}$, the stronger is the extinction of light by dust particles. The decrement intensities are defined by convention relatively to $H\beta$, $= I(H)/I(H\beta)$. An absolute flux calibration is unnecessary, because the units are anyway cancelled by the division. Particularly high is the effect of reddening in the Milky Way plane, on the Galactic latitude of ~0°. This relationship has already been detected by A. Shajn 1934.

For the determination of the "Interstellar reddening" with the Balmer decrement, appropriate objects are required, radiating the emission-lines of the H-Balmer series completely and not selectively attenuated - for example most of the emission nebulae and LBV stars. Representatives of the Mira Variables are unsuitable for the aforementioned reasons.

The following table shows by Brocklehurst [200] the theoretical decrement values $D_{th}$, quantum mechanically calculated for gases with a very low and high electron density $N_e$ and electron temperatures of $T_e$ 10000K and 20000K.

<table>
<thead>
<tr>
<th>Line</th>
<th>$D_{th} = I(H)/I(H\beta)$ for</th>
<th>$D_{th} = I(H)/I(H\beta)$ für</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>thin gas $(N_e = 10^2 \text{ cm}^{-3})$</td>
<td>dense gas $(N_e = 10^6 \text{ cm}^{-3})$</td>
</tr>
<tr>
<td></td>
<td>$T_e = 10000 \text{ K}$</td>
<td>$T_e = 20000 \text{ K}$</td>
</tr>
<tr>
<td>$H\alpha$</td>
<td>2.85</td>
<td>2.8</td>
</tr>
<tr>
<td>$H\beta$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$H\gamma$</td>
<td>0.47</td>
<td>0.47</td>
</tr>
<tr>
<td>$H\delta$</td>
<td>0.26</td>
<td>0.26</td>
</tr>
<tr>
<td>$H\epsilon$</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>$H8$</td>
<td>0.11</td>
<td>0.11</td>
</tr>
</tbody>
</table>

In the specialist jargon the two density cases are called "Case A" and "Case B".

**Case A:** A very thin gas, which is permeable for Lyman photons so they are enabled to escape the nebula.

**Case B:** A dense gas, which retains the short-wavelength ($\lambda < \text{Lyman }\alpha$) photons, which are therefore available for self-absorption processes.

The gas densities in emission nebulae are in the range between Case A and -B, by expanding stellar envelopes (Be- and LBV stars, Planetary Nebulae) Case B [33]. The electron temperature $T_e$ has in this range apparently only little influence on the decrement. The influence of the electron density $N_e$ is here noticeable only at the H\alpha line. Depending on the source, these values may differ in some cases.

Possible amateur applications are here rough decrement comparisons between different objects, eg various planetary nebulae, as well as Be- or LBV stars, located on different galactic latitudes.
19.4 Tests with the Balmer Decrement

Own experiments with the DADOS spectrograph (200L grating) yielded as expected steeper decrements. The line intensities \( I \) have been measured here as \( EW \)-values. The following table shows the EW values and decrements \( D \) determined at Gaussian fitted H-emissions lines in the normalised P Cygni profile of sect. 19.2. Earlier tests with higher-resolution profiles (900L/mm) yielded inconsistent and inaccurate results because the resolved P Cygni profiles did not allow proper Gaussian fits for the determination of the Equivalent Width EW. The following table shows the own measurements, compared with the theoretical values according to \( D_{Th} \) [200].

<table>
<thead>
<tr>
<th>Hydrogen line H</th>
<th>Measured values ( EW )</th>
<th>( D = I(H)/I(H\beta) )</th>
<th>Theoretical values ( D_{Th} = I(H)/I(H\beta) ) for Case B 20'000 K</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H\alpha )</td>
<td>-57.3</td>
<td>5.4</td>
<td>2.72</td>
</tr>
<tr>
<td>( H\beta )</td>
<td>-10.6</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( H\gamma )</td>
<td>-2.4</td>
<td>0.23</td>
<td>0.48</td>
</tr>
</tbody>
</table>

The profile of P Cygni shows, according to the Balmer decrement, a strong reddening. The accepted literature values vary between approximately 4.3 and 5.

19.5 Quantitative definition of the Balmer-decrement

For most astrophysical analysis the measured intensity ratio of the \( H\alpha \) and \( H\beta \) lines \( D_{obs} \) is required. This corresponds to the quantitative definition of the Balmer decrement:

\[
D_{obs} = I(H\alpha)/I(H\beta) \quad \{50\}
\]
20 Spectroscopic Determination of Interstellar Extinction

20.1 Spectroscopic Definition of the Interstellar Extinction

With the Balmer decrement $D$, the interstellar extinction $c$ can spectroscopically be determined. The extinction parameter $c$ characterises the entire extinction along the line of sight between the object and the observer. It is defined as the logarithmic ratio between the theoretical ($I_{th}$) and measured ($I_{obs}$) intensity of the $H\beta$ line [8]:

$$c(H\beta) = \log \frac{I(H\beta)_{th}}{I(H\beta)_{obs}} \quad \{51\}$$

$c(H\beta)$ is determined by the ratio between the measured and theoretical Balmer decrement $D_{obs}$ and $D_{th}$. The value 0.35 corresponds to the extinction factor $|f(\lambda)|$ at $\lambda = H\alpha$, according to the standard extinction curve after Osterbrock (chart below) [201].

$$c(H\beta) = \log \frac{D_{obs}}{D_{th}} \cdot \frac{1}{0.35} \quad \{52\}$$

In the context of such calculations in the literature the value $D_{th} = 2.86$ (Case $B$) has been established for the theoretical Balmer decrement. $D_{th} = 2.86$ inserted in (52) yields:

$$c(H\beta) = 2.86 \cdot \log D_{obs} - 1.31 \quad \{52a\}$$

20.2 Extinction correction with the measured Balmer decrement

The extinction is not constant but depends on the wavelength. With the correction function [10], the emission lines $I(\lambda)$ are adapted relatively to $I(H\beta)$ ("Dereddening").

$$I(\lambda) / I(H\beta) = I(\lambda)_{obs} / I(H\beta)_{obs} \cdot 10^{c(H\beta) \cdot f(\lambda)} \quad \{53\}$$

$f(\lambda)$ is defined as follows, where $A$ is the extinction at $A(\lambda)$ and $A(H\beta)$.

$$f(\lambda) = \frac{A(\lambda)}{A(H\beta)} - 1 \quad \text{in which } A(H\beta) = 1 \quad \{53a\}$$

Thus, the measured intensities $I(\lambda)_{obs}$ are reduced for $\lambda > H\beta$ and raised for $\lambda < H\beta$ (note the sign of $f(\lambda)$)! The value of $f(\lambda)$ is determined with an extinction curve, which exists in different versions with slightly different values. For amateur applications intermediate values may be roughly interpolated. Bottom left is the galactic standard extinction curve from Osterbrock (1989) with $f(\lambda)$ values 238. The table values to the right are from Seaton (1960).

<table>
<thead>
<tr>
<th>$\lambda$ (Å)</th>
<th>$f(\lambda)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3500</td>
<td>+0.42</td>
</tr>
<tr>
<td>4000</td>
<td>+0.24</td>
</tr>
<tr>
<td>4500</td>
<td>+0.10</td>
</tr>
<tr>
<td>4861</td>
<td>0.00</td>
</tr>
<tr>
<td>5000</td>
<td>-0.04</td>
</tr>
<tr>
<td>6000</td>
<td>-0.26</td>
</tr>
<tr>
<td>7000</td>
<td>-0.45</td>
</tr>
<tr>
<td>8000</td>
<td>-0.60</td>
</tr>
</tbody>
</table>

Galactic Extinction Law from Osterbrock 1989

From Seaton's 1960
20.3 Balmer Decrement and Color Excess

The measured Balmer decrement $D_{\text{obs}}$ also determines the color excess $E(\beta - \alpha)$ in [mag] for the Balmer lines and thus enables the link to the "classical" photometry [10].

$$E(\beta - \alpha) = -1.137 + 2.5 \cdot \log D_{\text{obs}} \quad (54)$$

If $D_{\text{obs}} = D_{\text{Th}} = 2.86$ follows: $E(\beta - \alpha) = 0$, in this special case, as expected, exists no reddening.

20.4 Application of Extinction Correction in the Amateur Sector

For most amateur spectroscopists, this correction is of secondary importance. It is anyway useful for the analysis of the emission nebulae (sect. 21), because the comparison of the diagnostic line intensities is affected by extinction. Furthermore the object itself provides here the emission lines, which are required for the determination of the decrement. But this purpose requires good-quality spectra of these faint objects. With exception of M42 this is a real challenge with average amateur means (see also [33]).

The faint, very small and almost star-like appearing Planetary Nebulae generally require relatively short exposure times. However they can only be recorded in the integrated light and barely selectively to a specific area within the nebula. Within these tiny appearing “slices”, larger intensity differences occur by the individual emission lines. Thus even the measured Balmer decrement may be distorted, aggravated by possible shifts in the slit position during recording, due to bad seeing or inadequate autoguiding. The most galactic Planetary Nebulae, reachable for Amateurs, show, with a few drastic exceptions (see sect. 21.11) values of $D \approx 2.9–3.5$ [10]. Therefore, this effect can usually be neglected for the rough determination of the excitation class, since the classification lines are quite close together (sect. 21.12).

In the professional sector, extinction corrections are performed with software support and are indispensable by the analysis of extragalactic emissionline objects. Even for objects within our neighbour galaxy M31, usually $D \gg 4$ [201].
21 Plasma Diagnostics for Emission Nebulae

21.1 Preliminary Remarks
In the “Spectroscopic Atlas for Amateur Astronomers” [33], a classification system is presented for the excitation classes of the ionised plasma in emission nebulae. Further this process is practically demonstrated, based on several objects. Here, as a supplement, further diagnostic possibilities are introduced, combined with the necessary physical background.

21.2 Overview of the Phenomenon “Emission Nebulae”
Reflection nebulae are interstellar gas and dust clouds which passively reflect the light of the embedded stars. Emission nebulae however are shining actively. This process requires that the atoms are first ionised by hot radiation sources with at least 25,000K. This requires UV photons, above the so-called Lyman limit of 912 Å and corresponding to an ionisation energy of >13.6 eV. This level is only achievable by very hot stars of the O- and early B-Class generating this way partially ionised plasma. By recombination the ions recapture free electrons which subsequently cascade down to lower levels, emitting photons of discrete frequencies $\nu$, according to the energy difference $\Delta E = h \cdot \nu$ (fluorescence effect). Thus, this nebulae produce by the fluorescence effect, similar to gas discharge lamps, mainly "quasi monochromatic" light, i.e. a limited number of discrete emission lines, which, with exception of the Supernova Remnants (SNR), are superimposed to a very weak emission-continuum. The energetic requirements are mainly met by $H\ II$ regions (e.g. M42), Planetary Nebulae PN (e.g. M57) and SNR (e.g. M1). Further mentionable are the Nuclei of Active Galaxies (AGN) and T-Tauri stars (sect. 17.2). The matter of the Nebulae consists mainly of hydrogen, helium, nitrogen, oxygen, carbon, sulphur, neon and dust (silicate, graphite etc.). Besides the chemical composition, the energy of the UV radiation, and the temperature $T_e$ as well as density $N_e$ of the free electrons characterise the local state of the plasma. This manifests itself directly in the intensity of emission lines, which simply allows a first rough estimation of important plasma -parameters.

21.3 Common Spectral Characteristics of Emission Nebulae
In all types of emission nebulae, ionisation processes are active even if with very different excitation energies. This explains the very similar appearance of the spectra. The diagram shows an excerpt from the spectrum of M42 with the two most striking features:

1. The intensity ratio of the brightest [O III] lines is practically constant with $I(5007)/I(4959) \approx 3$.
2. The Balmer decrement $D$. From the ratio between the measured and theoretical course the interstellar extinction can be determined (sect.20).

21.4 Ionisation Processes in $H\ II$ Emission Nebulae
These processes are first demonstrated schematically with a hydrogen atom. The high-energy UV photons from the central star ionise the nebula atoms and are thus completely absorbed, at latest at the end of the so-called Strömgren Sphere. Therefore here ends the partially ionised plasma of the emission nebula. Since the observed intensity of spectral lines barely varies, a permanent equilibrium between newly ionised and recombined ions must exist. Rough indicators for the strength of the radiation field are the atomic species,
the ionisation stage (sect. 11) and the abundance of the generated ions. The first two parameters can be directly read from the spectrum and compared with the required ionisation energy (see table below and [33]).

The kinetic energy $E_e$ of the electrons, released by the ionisation process, heats the nebula particles. $E_e$ corresponds to the surplus energy of the UV Photons, which remains after photo ionisation and is fully transformed to kinetic energy of the free electrons. The electron temperature $T_e$ and -density $N_e$ affect the following recombination- and collision excitation processes. $T_e$ is directly proportional to the average kinetic electron energy $E_e$ (Boltzmann constant $k_B = 8.6 \times 10^{-5} \text{eV/K}$).

$$E_e = k_B \cdot T_e \ [\text{eV}] \quad \{54\}$$

Formula (55) yields $E_e$ in Joule [$J$] with the electron mass $m_e = 9.1 \times 10^{-31} \text{kg}$ and $v$ [$\text{m/s}$]. The short formula (55a) gives $E_e$ directly in electron volts [eV].

$$E_e = \frac{1}{2} m_e \cdot v^2 \ [J] \quad \{55\} \quad E_e = 2.84 \cdot 10^{-12} \cdot v^2 \ [\text{eV}] \quad \{55a\}$$

### 21.5 Recombination Process

If an electron hits the ion centrally, it is captured and ends up first mostly on one of the upper excited levels (terms). The energy, generated this way is emitted as a photon $E_p$. It corresponds to the sum of the original kinetic energy of the electron $E_e$ and the discrete energy difference $\Delta E_n$ due to the distance to the Ionisation Limit. Since the share of the kinetic energy $E_e$ varies widely, from the recombination process a broadband radiation is contributed to the anyway weak continuum emission.

$$E_p = \frac{1}{2} m_e \cdot v^2 + \Delta E_n \quad \{56\}$$

### 21.6 Line Emission by Electron Transition

After recombination the electron “falls” either directly or via several intermediate levels $n$ (cascade), to the lowest energy ground state $n = 1$. Such transitions generate discrete line emission, according to the energy difference $\Delta E_n$. Most of these photons leave the nebula freely – including those which end in the pixel field of our cameras! This process cools the nebula, because the photons remove energy, providing thereby a thermal balance to the heating process by the free electrons. This regulates the electron temperature in the nebula in a range of ca. $5,000 \text{K} < T_e < 20,000 \text{K}$ [237].
21.7 Line Emission by Collision Excitation

If an electron hits an ion, then in most cases not a recombination but much more frequently a collision excitation occurs. If the impact energy is \( \geq \Delta E_n \), the electron is briefly raised to a higher level. By allowed transitions it will immediately fall back to ground state and radiate a photon of the discrete frequency \( \nu = \Delta E_n / h \), according to the energy difference.

Remark: Similarly, this process takes place in fluorescent lamps with low gas pressure. Due to the connected high tension, the electrons reach energies of several electron volts [eV], which subsequently excite mercury atoms to UV radiation. By contrast in dense gases, the excitation occurs mainly by collisions between the thermally excited atoms or molecules.

21.8 Line Emission by Permitted Transitions (Direct absorption)

In H II regions with O5 type central stars (eg M42) emission nebulae have Strömgren spheres with diameters of several light years, what extremely dilutes the radiation field. Thus, particularly in the extreme outskirts of the nebula, the probability gets extremely small, that the energy of a photon exactly fits to the excitation level of a hydrogen atom. Therefore the direct absorption of a photon doesn’t significantly contribute to the line emission. Further the main part of the photons is radiated in the UV range. Consequently many atoms are immediately ionised, once the energy of the incident photons is above the ionisation limit. Therefore a substantial line emission of permitted transitions is only possible by the recombination process. The high spectral intensity of Hydrogen and Helium, the main actors of the permitted transitions, is caused by the abundance which is by several orders of magnitude higher than the remaining elements in the nebula. The frequency of a specific electron transition also determines the relative intensity of the corresponding spectral line.

21.9 Line Emission by Forbidden Transitions

Emission nebulae contain various kinds of metal ions, most of them with several valence electrons on the outer shell. These cause electric and magnetic interactions, which multiplies the possible energy states. Such term schemes (or Grotrian diagrams) are therefore extremely complex and contain also so-called “Forbidden Transitions” (sect. 12). But the extremely thin nebulae provide ideal conditions, because the highly impact-sensitive and long-lasting metastable states become here very rarely untimely destroyed by impacts.

But first of all, these metal atoms must be ionised to the corresponding stage, which requires high-energy UV-photons. The required energies are listed in the following table, compared to hydrogen and helium [eV, \( \lambda \)]. The higher the required ionisation energy, the closer to the star the ions are generated (so called “stratification”) [10] [201].

<table>
<thead>
<tr>
<th>Ion</th>
<th>[S II]</th>
<th>[N II]</th>
<th>[O III]</th>
<th>[Ne III]</th>
<th>[O II]</th>
<th>H II</th>
<th>He II</th>
<th>He III</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E [\text{eV}] )</td>
<td>10.4</td>
<td>14.5</td>
<td>36.1</td>
<td>41.0</td>
<td>13.6</td>
<td>13.6</td>
<td>24.6</td>
<td>54.4</td>
</tr>
<tr>
<td>( \lambda [\text{Å}] )</td>
<td>1193</td>
<td>855</td>
<td>353</td>
<td>302</td>
<td>911</td>
<td>911</td>
<td>504</td>
<td>227</td>
</tr>
</tbody>
</table>

On the other hand The forbidden transitions of the metal ions need to populate the metastable initial terms from the ground state just a few electron volts [eV]. This small amount of energy is plentifully supplied by the frequent collision excitations of free electrons!
In comparison, for the permitted Hα line, at least 12.1 eV would be required from the ground state \((n1 - n3)\). For this, the electrons in the nebula are by far too slow, i.e. by about one order of magnitude (see diagram below). This explains the strong intensity of the forbidden-as compared to the allowed transitions. These metal ions are also called "cooler" [237] in the context of model computations. Influenced by the highly effective line emission they contribute significantly to the cooling of the nebula and therefore to the thermal equilibrium. The following chart shows just the relevant small excerpts of the highly complex term diagrams [10], [222]. For the most important metal ions, the required excitation energies and the wavelengths of the "forbidden" emissions are shown.

The following chart shows by Gieseking [222], the Maxwellian frequency distribution of electron velocities for relatively "cool" and "hot" nebulae, calculated for \(T_e\) 10,000K and 20,000K. Mapped are the two minimum rates for the excitation of the \([\text{O III}]\) lines. The upper edge of the diagram I have supplemented with the values of the kinetic electron energy. The maximum values of the two curves correspond to the average kinetic energy according to formula (54) (0.86eV and 1.72eV).
### 21.10 Scheme of the Photon Conversion Process in Emission Nebulae

This scheme summarises the previously described processes in H II regions and PN and illustrates important relationships. Not shown here are the bremsstrahlung processes, which become typically relevant just in SNR due to the relativistic electron velocities. Processes with photons are shown here in blue, those with electrons in red. So-called bound-bound transitions between the electron shells are marked with black arrows. The broad, gray arrows show the two fundamentally important equilibriums in the nebula:

- **The Thermal Equilibrium** in the nebulae between the heating process by the kinetic energy of free electrons and the permanent removal of energy by the escaping photons, regulates and determines the electron temperature $T_e$.

- **The Ionisation Equilibrium** between ionisation and recombination, regulates and determines the electron density $N_e$. If this balance is disturbed, the ionisation zone of the nebula either expands or shrinks [239].
21.11 Practical Aspects of Plasma Diagnostics

The main focus for amateurs is here the determination of the excitation class of emission nebulae. Their diagnostic lines are relatively intense; quite close together and therefore allow for galactic objects, even without extinction correction, a reasonable classification with accuracy of about $\pm 1$ class. Typical galactic decrement values are in the range of $D \approx 3.0$-$3.5$ [10]. However, there are stark outliers like NGC 7027 with $D \approx 7.4$ [10]! As already outlined in sect. 20.4, a special problem cause the faint, very small and almost star-like appearing Planetary Nebulae. Own tests have shown that by the same object the analysis of several spectra can show significantly different results and also the Balmer decrement may be affected (examples see [33]). Anyway the influence on the excitation class was observed as quite low!

The determination of additional plasma parameters such as the electron temperature $T_e$ and the electron density $N_e$ specifically requires low-noise spectra with high resolution, regarding these faint objects, a real challenge with amateur equipments. In addition, some of the used diagnostic lines are extremely weak and the error rate is correspondingly high. Even between values in professional publications, often large deviations are noted!

21.12 Determination of the Excitation Class $E$

Since the beginning of the 20th Century numerous methods have been proposed to determine the excitation classes of emission nebulae. The 12-level “revised” Gurzadyan system [10], which has been developed also by, Aller, Webster, Acker and others, is one of the currently best accepted and appropriate also for amateurs. It relies on the simple principle that with increasing excitation class, the intensity of the forbidden [O III] lines becomes stronger, compared with the H-Balmer series. Therefore as a classification criterion the intensity sum of the two brightest [O III] lines, relative to the H$\beta$ emission, is used. Within the range of the low excitation classes $E$: 1–4, this value increases strikingly. The [O III] lines at $\lambda\lambda 4959$ and 5007 are denoted in the formulas as $N_1$ and $N_2$.

For low excitation classes $E1 – E4$: $I_{N1+N2}/I_{H\beta}$

Within the transition class $E4$ the He II line at $\lambda 4686$ appears for the first time [225]. It requires 24.6 eV for the ionisation. That's almost twice the energy as needed for H II with 13.6 eV. From here on, the intensity of He II increases continuously and replaces the now stagnant H$\beta$ emission as a comparison value in the formula. The ratio is expressed here logarithmically (base 10) in order to limit the range of values for the classification system:

For middle and high Excitation Classes $E4 – E12$: $\log(I_{N1+N2}/I_{He\ II (4686)})$

The 12 $E$-Classes are subdivided in to the groups Low ($E = 1 – 4$), Middle ($E = 4 – 8$) and High ($E = 8 – 12$). In extreme cases $12^+$ is assigned.

<table>
<thead>
<tr>
<th>Low $E$-Class</th>
<th>$I_{N1+N2}/I_{H\beta}$</th>
<th>Middle $E$-Class</th>
<th>$\log(I_{N1+N2}/I_{4686})$</th>
<th>High $E$-Class</th>
<th>$\log(I_{N1+N2}/I_{4686})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E1$</td>
<td>0 – 5</td>
<td>$E4$</td>
<td>2.6</td>
<td>$E9$</td>
<td>1.7</td>
</tr>
<tr>
<td>$E2$</td>
<td>5 – 10</td>
<td>$E5$</td>
<td>2.5</td>
<td>$E10$</td>
<td>1.5</td>
</tr>
<tr>
<td>$E3$</td>
<td>10 – 15</td>
<td>$E6$</td>
<td>2.3</td>
<td>$E11$</td>
<td>1.2</td>
</tr>
<tr>
<td>$E4$</td>
<td>&gt;15</td>
<td>$E7$</td>
<td>2.1</td>
<td>$E12$</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$E8$</td>
<td>1.9</td>
<td>$E12^+$</td>
<td>0.6</td>
</tr>
</tbody>
</table>
21.13 The Excitation Class as an Indicator for Plasma Diagnostics

Gurzadyan (among others) has shown that the excitation classes are more or less closely linked to the evolution of the PN [10], [226]. The study with a sample of 142 PN showed that the E-Class is a rough indicator for the following parameters; however in reality the values may scatter considerably [8].

1. The age of the PN
   Typically PN start on the lowest E- level and subsequently step up the entire scale with increasing age. The four lowest classes are usually passed very quickly. Later on this pace decreases dramatically. The entire process takes finally about 10,000 to >20,000 years, an extremely short period, compared with the total lifetime of a star!

2. The Temperature $T_{\text{eff}}$ of the central star
   The temperature of the central star also rises with the increasing E-Class. By pushing of the envelope, increasingly deeper and thus hotter layers of the star become "exposed". At about E7 in most cases an extremely hot White Dwarf remains, generating a WR-like spectrum. For $T_{\text{eff}}$ [K] the following, very rough estimates can be derived [33]:

<table>
<thead>
<tr>
<th>E-Class</th>
<th>E1-2</th>
<th>E3</th>
<th>E4</th>
<th>E5</th>
<th>E7</th>
<th>E8-12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{\text{eff}}$ [K]</td>
<td>35,000</td>
<td>50,000</td>
<td>70,000</td>
<td>80,000</td>
<td>90,000</td>
<td>100,000 – 200,000</td>
</tr>
</tbody>
</table>

3. The Expansion of the Nebula
   The visibility limit of expanding PN lies at a maximum radius of about 1.6 ly (0.5 parsec). With increasing E-class, also the radius of the expanding nebula is growing. Gurzadyan [226] provides mean values for $R_n$ [ly] which however may scatter considerably for the individual nebulae.

<table>
<thead>
<tr>
<th>E-Class</th>
<th>E1</th>
<th>E3</th>
<th>E5</th>
<th>E7</th>
<th>E9</th>
<th>E11</th>
<th>E12+</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_n$ [ly]</td>
<td>0.5</td>
<td>0.65</td>
<td>0.72</td>
<td>1.0</td>
<td>1.2</td>
<td>1.4</td>
<td>1.6</td>
</tr>
</tbody>
</table>

21.14 Estimation of $T_e$ and $N_e$ with the O III and N II Method

Due to the very weak diagnostic lines, these methods can be applied only to spectra with high resolution and quality. Further J. Schmoll outlines in his dissertation [201], what influence the slit width and the method of background subtraction exert on the analysis of weak lines! The procedure is based on the fundamental equations by Gurzadyan 1997 [10], which uses for the O III method the lines at $\lambda\lambda$ 5007, 4959 and 4363, and for the N II method at $\lambda\lambda$ 6548, 6584 and 5755.

**O III Procedure:**

$$\frac{I(5007) + I(4959)}{I(4363)} = 0.0753 \cdot \frac{1 + 2.67 \cdot 10^5 \cdot \sqrt{T_e} / N_e}{1 + 2.3 \cdot 10^3 \cdot \sqrt{T_e} / N_e} \cdot e^{33000/T_e} \quad \{57\}$$

**N II Procedure:**

$$\frac{I(6548) + I(6584)}{I(5755)} = 0.01625 \cdot \frac{1 + 1.94 \cdot 10^5 \cdot \sqrt{T_e} / N_e}{1.03 + 3.2 \cdot 10^2 \cdot \sqrt{T_e} / N_e} \cdot e^{25000/T_e} \quad \{58\}$$

For the calculation of the electron temperature, these equations can’t be explicitly converted and solved by $T_e$ and contain additionally the variable $N_e$. But for $T_e$, empirical formulas exist, which are valid for thin gases $N_e < 10^3 \text{ cm}^{-3}$ (typical for H II regions and SNR). “ln” is the natural logarithm to base e.
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\[ O \text{ III Procedure:} \quad T_e = \frac{33'000}{\ln \left( R_1/8.74 \right)} \quad \{59\} \quad \text{in which} \quad R_1 = \frac{I(5007) + I(4959)}{I(4363)} \]

\[ N \text{ II Procedure:} \quad T_e = \frac{25'000}{\ln \left( R_2/9.85 \right)} \quad \{60\} \quad \text{in which} \quad R_2 = \frac{I(6548) + I(6584)}{I(5755)} \]

For the explicit calculation of \( N_e \), with known \( T_e \), I have converted the formulas \{57\} \{58\} accordingly:

\[ O \text{ III Procedure:} \quad N_e = \sqrt{T_e} \cdot \frac{2.67 \cdot 10^5 - 3.05 \cdot 10^4 \cdot R_1}{13.3 \cdot R_1 \left( e^{33000/T_e} \right) - 1} \quad \{61\} \]

\[ N \text{ II Procedure:} \quad N_e = \sqrt{T_e} \cdot \frac{1.94 \cdot 10^5 - 1.97 \cdot 10^4 \cdot R_2}{61.5 \cdot R_2 \left( e^{25000/T_e} \right) - 1} \quad \{62\} \]

If the recording of a spectral profile can be limited on a defined region within the nebula, in both equations \{61\} and \{62\} the \( N_e \) variables become identical. \( N_e \) can then be eliminated by equalisation of \{61\} and \{62\}. The implicitly remaining variable \( T_e \) anyway requires finally an iteratively solving of the equation. However this requires that the values of all diagnostic lines, for both methods, are available in good quality.

21.15 Estimation of Electron Density from the S II and O II Ratio

The electron density \( N_e \) can be estimated by Osterbrock from the ratio of the two sulfur lines \( S \ II \ \lambda \lambda \ 6716, 6731 \) or the oxygen lines \( O \ II \ \lambda \lambda \ 3729, 3726 \) [201]. The big advantage of this method: These lines are so close together, that the extinction can’t exert any significant effect on the ratio. The disadvantage is that the two lines, except by SNR, are very weak and therefore difficult to measure.

\[ \text{For S II:} \quad \frac{I(6716)}{I(6731)} \]
\[ \text{For O II:} \quad \frac{I(3729)}{I(3726)} \]

![Intensity Ratio vs Electron Density](image)
21.16 Distinguishing Characteristics in the Spectra of Emission Nebulae

Due to the synchrotron and bremsstrahlung SNR show, especially in the X-ray part of the spectrum, a clear continuum. X-ray telescopes are therefore highly valuable to distinguish SNR from the other nebula species, particularly by very faint extragalactic objects. For all other types of Emission Nebulae the detection of a continuum radiation is difficult.

In the optical part of SNR spectra, the [S II] and [O I] lines are, relative to Hα, more intense than at PN and H II regions, due to additionally shock wave induced collision ionisation. The [S II] and [O I] emissions are very weak at PN and almost totally absent in H II regions [201]. The electron density $N_e$ is very low in SNR, ie somewhat lower than in H II regions. It amounts in the highly expanded, old Cirrus Nebula to about 300 cm$^{-3}$:

By the still young and compact Crab Nebula it is about 1000 cm$^{-3}$ [201]. By PN, $N_e$ gets highest and is usually in the order of $10^4$ cm$^{-3}$ [201]. In the H II region of M42, $N_e$ is within the range of 1000–2000 cm$^{-3}$ [224].

In H II regions, the excitation by the O- and early B-class stars is relatively low and therefore the excitation class in the order of E = 1-2 only modest.

Planetary nebulae usually pass through all 12 excitation classes, following the evolution of the central star.

In this regard the SNR are also a highly complex special case. By very young SNR, eg the Crab Nebula (M1), dominate higher excitation classes whose levels are not homogeneously distributed within the nebula, according to the complex filament structure [231]. The diagnostic line He II at $\lambda$ 4686 is therefore a striking feature in some spectra of M1, [231].
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